Reinvestment Flows under Leverage and Endogenous Growth

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Abstract

A firm growing endogenously and wishing to keep a constant leverage ratio will be able to invest not only the retained earnings but also the amount of new issued debt. Therefore, when leverage is accounted for reinvestment flows differ from retained equity holder earnings. The link between leverage, dividend payout, reinvested flows and the growth rate are explored and it is verified that reinvested flows are bounded and that they rise when dividend payouts fall, in such a way that the higher the leverage the higher the invested flows for each level of dividend payout.

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Introduction

In most firm valuations a terminal value is assumed to account for the value generated after the forecasting period. Whenever the terminal value is the present value of a growing perpetuity the magnitude of the growth rate is by far the key variable.

The growth rate is closely tied to the firm's investment opportunities and its magnitude depends basically on the proportion of free cash flows to be retained for reinvestment and on the rate of return on this investment.

Although these concepts are well-known, little attention is generally given to the impact of funding flows on the growth rate.

From the equity side the proportion of retained (equity holder) earnings is just what is left after dividends are paid out. However, a firm growing endogenously and wishing to keep a constant leverage ratio will be able to invest not only the retained earnings but also the amount of new debt to be issued that is associated with these new retained earnings. Therefore, reinvestment flows are determined not only by the dividend payout but by the debt ratio as well.

The purpose of this paper is to find the links among leverage, the dividend payout, reinvested cash flows and the firm's growth rate under endogenous growth and a constant leverage ratio. In section 2 the general background is reviewed. In section 3 the relationships between the mentioned variables are developed. In section 4 a practical example is shown. The last section contains the conclusions.[1]

[1] I am indebted to Eduardo Pablo, Jordi Fabregat, Jesús Palau, Luisa Alemany, Josep Alvarez de Lara and Santiago Forte for helpful comments.

General Background

Firm valuation under the discounted cash flow rule requires the projection of "Free Cash Flows" along a forecasting period. The free cash flow (FCF) is defined as the total cash flow available to investors, both debt holders and equity holders, and is often computed as:

$$FCF = EBIT + DA - T - Capex - INWC \tag{1}$$

Where:

EBIT : Earnings before interest expenses and taxes
DA : Depreciation and amortization
T : Corporate taxes[2]
Capex: Net capital investment (fixed assets)
INWC : Investment in net working capital

In firm valuations a terminal value is usually specified at the end of the free cash flow forecasting period. This terminal value is generally assumed to be either a liquidation value or the present value of a growing or no-growing perpetuity.[3]

A no-growing perpetuity implies that the free cash flow at the end of the horizon is repeated forever. In this instance, the firm's investment in fixed assets period after period exactly matches the depreciation and amortization allowance, and investment in net working capital is null. That is:

$$\begin{aligned} DA &= Capex \\ INWC &= 0 \end{aligned} \tag{2}$$

[2] T = (EBITDA-DA)T_C where T_C corresponds to the corporate tax rate
[3] The terminal value is occasionally made equal to the value of the firm based on some kind of multiple such as a number of times annual sales or *EBITDA*.

Therefore:

$$FCF = EBIT - T \tag{3}$$

This expression is known as: "Net Operating Profit After Taxes" (NOPAT) and corresponds to total cash flows received by fund providers (debt holders and equity holders).[4]

For the free cash flow to grow in perpetuity, starting at the end of the forecasting period the firm must maintain forever a positive net investment in fixed assets (beyond depreciation and amortization) and/or net working capital. Therefore, the net stock of fixed assets and/or net working capital will keep growing in perpetuity.

This investment can be financed exogenously, endogenously or through a combination of both. Investment is exogenous when sourced from new equity and debt issues. Investment is endogenous when financed from the firm's free cash flows.

This paper focuses on the terminal value as the present value of a growing perpetuity under endogenous growth, meaning that no new equity is issued.

Endogenous growth is feasible only by periodically reinvesting part of NOPAT. These funds are reinvested at the expected return of invested capital after taxes r.

Besides the expected return on invested capital r, there are two other rates pertaining the return on invested capital: the historical return on invested capital (ROIC) and the minimum expected return on assets also known as the "hurdle rate" (HR).[5]

[4] Actually *NOPAT* only approximates the cash flows to fund providers since it does not account for accrued items such as deferred taxes, prepaid insurance etc.
[5] The hurdle rate is often assumed to be equal to the Weighted Average Cost of Capital (*WACC*).

It is important to differentiate among these three rates: ROIC is an ex-post concept since it refers to past results whereas the other two rates look to the future and are ex-ante. On the other hand, r must be greater than or equal than HR since this rate is the minimum return demanded by investors while r is the return expected by investors.

For the firm to add value consistently the return on invested capital r must always be larger than HR. Notice that this does not mean that the ex-post return on invested capital ROIC will always be larger than HR, only that investors demand the firm to invest capital at a rate r larger than HR.

However, in practice it is expected that the firm's competitive advantages and investment opportunities will erode through time. Thus, it is unlikely for the firm to invest at a rate above the HR in perpetuity. Hence, when assuming a reinvested rate r (larger than the HR) in perpetuity this paper is focusing on the maximum possible level of endogenous growth.

Leverage, Dividend Payout and Reinvested Cash Flows

Following, the link between reinvested funds, leverage and the dividend payout will be explored. All variables are based on accounting values.

Let us define:

EAT : Expected Earnings After Taxes for the first year after the forecasting period,
p : Dividend payout, which is the portion of EAT to be distributed as dividends
Thus, the proportion of retained earnings by equity holders (or "plow back ratio") will be:

$$1 - p \tag{4}$$

An indebted firm wishing to issue debt in a constant proportion to Net Worth will be able to invest not only the future retained earnings but also the amount of new debt to be issued that is associated with future retained earnings.

Let us define b as the "retention ratio". b is the portion of future NOPAT, adjusted by the impact of new issued debt, to be reinvested. It is determined both by the expected dividend payout p and the leverage ratio D/NW.

Let us define:

D: Total debt at the end of the first year after the horizon
r_D : Long-term interest rate on debt (assumed constant)
NW: Net Worth at the end of the first year after the horizon
The expected cash flow to debt holders during the first year after the horizon is given by:

$$r_D \cdot D \tag{5}$$

Therefore, the total after-tax expected cash flow to investors for the first year after the horizon (or expected NOPAT) will be:

$$EAT + r_D \cdot D \tag{6}$$

Expected retained equity holder earnings are given by:

$$EAT \cdot (1 - p) \tag{7}$$

If a constant (accounting based) leverage ratio is to be maintained, the new matching debt must be added to retained earnings:

$$EAT \cdot (1-p) \cdot \left(1 + {}^{D}\!/_{NW}\right) \tag{8}$$

This is the total expected invested capital as a function of the payout ratio p and the leverage ratio D/NW.

The retention ratio b equals expected invested capital as a proportion of forecasted free cash flow (or NOPAT). Therefore:

$$b = \frac{EAT \cdot (1-p) \cdot \left(1 + {}^{D}\!/_{NW}\right)}{EAT + r_D \cdot D} \tag{9}$$

The firm's growth rate g will be given by:

$$g = b \cdot r \tag{10}$$

For the present value of a growing perpetuity to have economic meaning the growth rate must always be smaller than both HR and long-term economic growth (G). Hence, the following constraint must hold:

$$br = g < Min(HR, G) \Rightarrow b < \frac{Min(HR, G)}{r} \tag{11}$$

A Practical Example

For illustration, a practical example of the computation of b for a leveraged firm under endogenous growth follows. Let us assume this data:

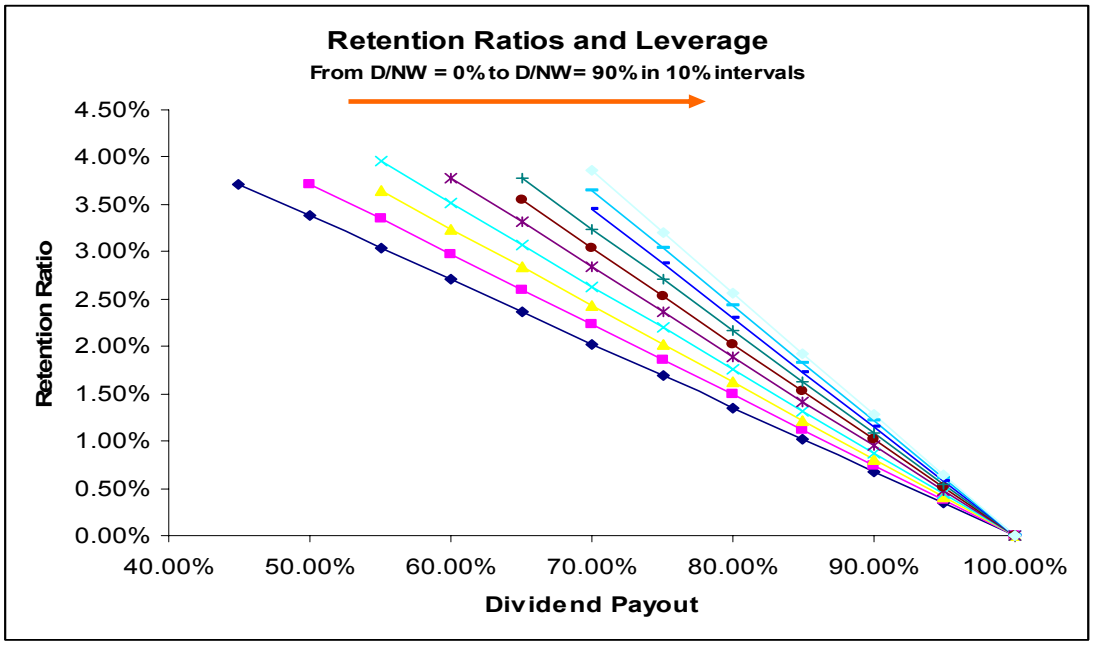
Data		
Dividend Payout	p	50.00%
Discount Rate on Debt	r_D	6.00%
Hurdle Rate	HR	8.00%
Long-Term Economic Growth	G	4.00%
Excess Return on Invested Capital	r-HR	2.00%
Return on Invested Capital	r	10.00%
Computations		
Target Leverage Ratio	D/NW	20.00%
Total Debt on Proj. Earnings	D/EAT	8.00
Retention Ratio	b	40.54%
Growth Rate	g	4.05%

In the following table it can be observed how for a fixed leverage ratio, b and g increase as p decreases. Note how for this set of parameters values of p below 55% are unfeasible since they would imply g to overtake either HR or G.

	D/NW =	**20.00%**
p	**b**	**g**
100.00%	0.00%	0.00%
95.00%	4.05%	0.41%
90.00%	8.11%	0.81%
85.00%	12.16%	1.22%
80.00%	16.22%	1.62%
75.00%	20.27%	2.03%
70.00%	24.32%	2.43%
65.00%	28.38%	2.84%
60.00%	32.43%	3.24%
55.00%	36.49%	3.65%
50.00%	40.54%	ERROR
45.00%	44.59%	ERROR
40.00%	48.65%	ERROR
35.00%	52.70%	ERROR
30.00%	56.76%	ERROR
25.00%	60.81%	ERROR
20.00%	64.86%	ERROR
15.00%	68.92%	ERROR
10.00%	72.97%	ERROR
5.00%	77.03%	ERROR
0.00%	81.08%	ERROR

In the diagram below it can be seen how retention ratios change with dividend payouts for different leverage ratios. Be aware that the

higher the leverage the higher the retention ratios for each level of payout. Nonetheless, retention ratios are once more bounded by the constraint b<[Min (HR,G)]/r and are variable depending on the value of the defining parameters.



Conclusions

The terminal value assumed in most firm valuations is the present value of a growing perpetuity. The growth rate depends basically on the proportion of free cash flows to be retained for reinvestment and on the rate of return on this investment.

A firm growing endogenously and wishing to keep a constant leverage ratio will be able to invest not only the retained earnings but also the amount of new issued debt that is associated with these new retained earnings. Therefore, when leverage is accounted for reinvested funds differ from retained equity holder earnings.

In this paper the link in accounting terms between leverage, dividend payout, reinvested flows and the growth rate was explored for the maximum possible level of endogenous growth. It was verified that: a) for fixed leverage ratios and reinvestment rates reinvested flows and the growth rate increase as the dividend payout decreases; b) reinvested flows rise when dividend payouts fall in such a way that the higher the leverage the higher the reinvested flows for each level of dividend payout and; c) reinvested flows are bounded (because growth rates cannot be larger than either the hurdle rate or long-term economic growth) and vary depending on the value of the defining parameters.