The Discount Rate of a Debt is not always the Cost of Debt
Technical Note

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Abstract

Assuming that the discount rate for valuing a debt equals the debt's yield is an approach that is inconsistent with the CAPM model. The debt's yield corresponds to its internal rate of return whereas, according to the CAPM, the discount rate should only reflect the systematic risk associated with the debt's returns. Furthermore, in general, default and liquidity risks will have both a systematic and an unsystematic component. Therefore these risks may only have a partial impact on the discount rate, which should only be affected by their systematic component.

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Introduction

It is common practice to assume that the discount rate for valuing a debt equals the debt's yield. This paper asserts that this practice is inconsistent with the CAPM model whenever significant default or liquidity risks are present.

The inconsistency stems from the fact that the debt's yield does not only reflect systematic risk but also lack of liquidity and the debtor's probability of default. The debt's yield corresponds to its internal rate of return, whereas the discount rate should only reflect the systematic risk associated with the debt's returns.

For simplicity, our argumentation will be illustrated with the following examples:[1]

Zero Systematic Risk Zero-Coupon Bonds

Let us start with a simple risk-free zero coupon bond:[2]

Bond 1:

Assume a \$1,000 zero coupon bond maturing in one year. Say that the bond is very liquid and risk-free and that the one-year risk-free rate is 5 %. The present value of bond 1 PV_1 will be:

$$PV_1 = \frac{1000}{1.05} = \$952.38 \tag{1}$$

Now, the possibility of default is brought in,

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[2] For simplicity, commissions and taxes are ignored throughout the examples.

Bond 2:

Say that now bond 1 while still highly liquid has a 20% probability of default. Let us further assume (rather unrealistically) that all its risk, including default risk, is completely uncorrelated with the market portfolio. Following the CAPM, this means that the bond's beta is zero and that it must be discounted at the risk-free rate. The present value of bond 2 PV_2 will be:

$$PV_2 = \frac{(0.8 \cdot 1000 + 0.2 \cdot 0)}{1.05} = \$761.91 \tag{2}$$

There are two types of return associated with bond 2: *promised* and *expected*.

The first is the return pledged by the issuer to the debt holder. It supposes that all contractual cash flows will actually be paid. The second return allows for the possibility that the debtor's promise will not be fulfilled. Here, there is no guarantee that the contractual cash flows will actually take place and there is a significant probability (20 %, in our example) that the debt holder will end up with a zero cash flow at maturity.

For bond 2, the *promised return* is no other than the bond's internal rate of return (IRR) or yield to maturity (YTM):

$$\frac{1000}{761.91} - 1 = 31.25\% \tag{3}$$

Whereas bond 2's *expected return* factors in the probability of default:

$$\frac{(0.8 \cdot 1000 + 0.2 \cdot 0)}{761.91} - 1 = 5\% \tag{4}$$

Observe that the expected return is actually the discount rate k_D.[3]

Were bond 2 illiquid, investors would further decrease expected cash flows to account for the liquidity-related transaction costs of buying and selling the bond in the market. The effect on expected cash flows would be quite similar to that of default risk.

Positive Systematic Risk Zero-Coupon Bonds

Now, let us assume that the bond is still highly liquid, but default risk is totally or partially systematic and therefore the bond has a positive beta.

Bond 3:

Suppose that besides having a 20 % probability of default the bond has a beta of 0.2.

Assuming a 4.5 % market risk premium and applying the CAPM the discount rate for bond 3 will be:

$$k_D = 5\% + 0.2 \cdot 4.5\% = 5.9\% \tag{5}$$

And its present value:

$$PV_3 = \frac{\left(0.8 \cdot 1000 + 0.2 \cdot 0\right)}{1.059} = \$755.43 \tag{6}$$

The promised return (YTM) for bond 3 will be:

$$\frac{1000}{755.43} - 1 = 32.37\% \tag{7}$$

[3] Bond 2 having no systematic risk, the objective (i.e. real), probabilities affecting the expected cash flow correspond in this example to risk-neutral probabilities.

And the expected return (k_D) will rise to 5.9%.

Comparing this with bond 2, promised return increased from 31.25 % to 32.37 %, and expected return from 5 % to 5.9 %. However, these increases are due exclusively to a larger systematic risk since the probability of default remains unchanged for both bonds.[4]

Again, if bond 3 were illiquid investors would decrease expected cash flows even more to account for the added transaction costs. The effect on expected cash flows would be much like that of default risk; and the effect on the discount rate would similarly depend on how systematic liquidity risk turned out to be.

It can be concluded that even if default/liquidity risks are totally or partially systematic, for risky bonds there will always be a significant difference between promised return (i.e. yield YTM) and expected return (i.e. discount rate k_D).

Discount Rates, Yields and Bond Prices

The discount rate k_D and yield YTM are one and the same only when the probability of default and lack of liquidity are negligible, and so expected and promised cash flows are practically identical.

A bond's present value can be computed either by discounting promised cash flows at YTM or by discounting expected cash flows at k_D. The more significant the default/liquidity risks, the larger the gap between YTM and k_D; and this is true independently of whether default/liquidity risks are more or less systematic.

In reality, bonds are not as simple as those in our examples. They can have coupons and different maturities. Also, default/liquidity risks and

[4] Unlike bond 2, bond 3 has some systematic risk. Hence the objective probabilities affecting the expected cash flow do not match the risk-neutral probabilities in this case.

recovery rates may improve or deteriorate over time. Therefore, analysts build their expectations not on a simple pay/no-pay situation, but on changing recovery rate scenarios and complex probability distributions that are continuously revised as information about the firm reaches the market. Nonetheless, however complex the analysis process, the rationale remains the same:

- default/liquidity risks affect contractual cash flows throughout the life of the bond;
- default/liquidity risks may be more or less systematic, more or less affecting the discount rate , and
- the bond's price is set according to the market's assessment of both the impact of default and illiquidity on expected cash flows, and the extent to which systematic risk affects the discount rate.

In all instances, it remains clear that promised (YTM) and expected returns (k_D) will differ whenever perceived default/liquidity risks are significant.

YTM is directly observable. When the debt is actively traded, YTM is just its IRR. When the debt is not actively traded, YTM can be made equal to the returns of debts with a similar risk rating.

Nonetheless, unlike YTM, k_D is not directly observable. Whenever YTM and k_D are not equal, k_D can be estimated by performing a regression between the historical returns of the debt and the historical returns of a proxy for the market portfolio (for instance, the S&P 500 index). The regression coefficient will be an estimator for β_D, and k_D will be the CAPM return rate corresponding to $\beta_{D.}$[5]

[5] Unfortunately, the computation of the debt's beta through the CAPM presents practical problems. In particular, bond betas are not stable but depend on interest rate shifts and the time to maturity of the bond. The longer the time to maturity, the more sensitive will the bond's price be to interest rate shifts, and the higher its beta.

Default Risk: The Case of Emerging Markets

It is often assumed in the finance literature that the beta of the debt is zero (or close to zero) only when leverage increases the debt's beta and discount rate rise. The rationale is that for low leverage ratios, asset value is so high in comparison to the debt level that for all practical purposes the probability of default is nil.

However, as the debt ratio rises, the debt approaches asset value. Hence any significant drop in asset value impairs the ability to serve the debt, increasing the probability of default. For very high leverage, the debt starts to mirror asset risk and its beta approaches the beta of the unleveraged firm.

This line of thinking presumes that the probability of default stems only from a firm's risk and that it has no connection with other sources of risk foreign to the firm. Nevertheless, this is not often the case in emerging markets, where corporate debt ratings are closely tied to country risk. In fact, international rating agencies only rarely assign credit ratings to the debt of local firms above that of local government debt, under the presumption that no entity can be less risky than the government of the country where it operates.

Observe that this reasoning factors in a source of risk different from the firm itself. This new source is country risk, as reflected in the yield of government debt. But country risk often has a considerable political component, since many developing country governments make decisions affecting business that are politically motivated, with little if anything to do with market risk. In other words, country risk is often to a considerable extent non-systematic.

In conclusion in emerging markets the perceived default risk for local firm debts is not exclusively associated with leverage and firm value. Thus, it could have a significant non-systematic component and not be completely reflected on the discount rate.

A Real Life Example

In the following real life example it will be shown how discount rate and yield might differ in an emerging market. The chosen bond is:

Firm	Petrobras International (Brazil)
Type	Notes
Currency	US$
Issued	Sep 11, 2003
Maturity	Jul 2, 2013
Interest Rate	9.125 % yearly fixed
Coupons	Twice yearly on Jan 2 and Jul 2

On December 2, 2004 the bond's (coupon) adjusted price and yield were 116.033 and 7.191 %, respectively.

Using the Standard & Poor's 500 Composite Index as the proxy for the market portfolio, the beta of the bond was estimated to be 0.054. Based on a market risk premium of 4.5% and a risk-free rate of 4.25 %[6] the expected return for the bond on December 2 2004 is 4.49 %.

For this expected return the discounted cash flows result in a present value of 141.53. Observe how this value contrasts with the (much lower) bond adjusted price of 116.033. This means an 18 % reduction in value due to expected default/liquidity risks.

The Discount Rate of Debt, WACC and the CAPM

It is worthwhile to point out a common inconsistency often found in many real life valuations.

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[6] For the market risk premium, refer to Dimson, Marsh & Stauton, 2003, "Global Evidence on the Equity Risk Premium", *Journal of Applied Corporate Finance*, vol. 15, no. 4. The risk-free rate corresponds to the yield on 10 year US-T Bonds on December 2, 2004.

Project betas are often unleveraged by using the formula:

$$\beta_u = \frac{\beta_E}{\left[1 + \left(1 - T_C\right) D / E\right]} \tag{8}$$

where,

β_E is the equity beta

β_u is the unleveraged beta

T_C is the corporate tax rate

D is the market value of debt

E is the market value of equity

This formula is just a simplified expression of:

$$\beta_u = \frac{\beta_E + \beta_D \left(1 - T_C\right) D / E}{\left[1 + \left(1 - T_C\right) D / E\right]} \tag{9}$$

where it is understood that beta of the debt β_D is zero.

When unleveraging beta in this way, practitioners frequently make k_D as equivalent to YTM, or alternatively to the return associated with the firm's risk rating. The lower the risk rating the higher the discount rate and vice-versa.

It is clear that if the CAPM is the model in use there is a fundamental contradiction in this approach. By making the discount rate equal to YTM, the full effect of default/liquidity risk on the valuation (whether systematic or not) is integrated into the discount rate, whereas the CAPM model requires discount rates to be affected only by systematic risk.

If β_D is zero for unleveraging purposes it must also remain zero for the computation of k_D. Hence k_D must equal the risk free rate. Otherwise, beta cannot be unleveraged with formula (8) but with formula (9) by inserting a value for β_D consistent with k_D.

Having said this, it is true that making YTM equal to k_D is a practical way to incorporate the costs of financial distress in the discount rate. Though inconsistent with the CAPM, this practice will be appropriate as long as its impact on the valuation accurately matches, on the one hand, the effect of the costs of financial distress on expected cash flows and, on the other hand, the impact on the discount rate of the systematic risk stemming from the costs of financial distress.[7,8]

Conclusions

Assuming that the discount rate for valuing a debt equals the debt's yield is inconsistent whenever the CAPM is the model employed for discount rate determination.

The inconsistency stems from the fact that the debt's yield does not only reflect systematic risk but also lack of liquidity and the debtor's probability of default. The debt's yield corresponds to its internal rate of return, whereas the discount rate should only reflect the systematic risk associated with the debt's returns.

There are two types of return associated with default/illiquidity prone bonds: promised and expected returns. Promised return considers that all contractual cash flows will be actually paid. Expected return allows

[7] The WACC formula is, in itself, also inconsistent with the CAPM: when adjusting the WACC rate with the benefit of the tax shield, the full effect of the tax shield on firm value is built into the discount rate without taking into consideration the extent to which this adjustment reflects systematic risk.

[8] This practice is also frequently employed to find the optimal capital structure at the point where the WACC rate is minimized.

for the possibility that the debtor's promise will not be fully complied with.

Promised return corresponds to the bond's yield, whereas expected return corresponds to the bond's discount rate. The two are equal only when default/liquidity risks are nil and expected and promised cash flows are identical.

Whatever the degree to which default/liquidity risks might be systematic, there will always be a significant difference between the discount rate and the yield for bonds perceived to be risky, meaning those issued by emerging-market or below-investment grade firms.

Having said this, making the discount rate of debt equal to its YTM is a practical (though possibly inaccurate) way to incorporate the costs of financial distress in the WACC discount rate.