A Parsimonious Approach to Interaction Effects in Structural Equation Models: An Application to Consumer Behavior

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Received: November, 2003

Abstract

Structural equation models with non-linear constraints make it possible to estimate interaction effects while correcting for measurement error. Up to now, only direct effects have been specified, thus wasting much of the capability of the structural equation approach. In addition, the complexity of this approach has made its use much less appealing than it deserves in the marketing and management literatures. This article questions the actual usefulness of the constraints in the current specifications, and proposes reducing their number or even eliminating them completely, which leads to a more easily handled model that is also more robust to non-normality. The paper also presents and discusses an extension of Jaccard and Wan's, and Jöreskog and Yang's specification that can handle direct, indirect and interaction effects simultaneously. The approach is illustrated using empirical data for studying indirect and moderate effects of "value orientations with respect to the environment" on both "possibilities of influencing events" and on "environmentally-friendly consumer behavior" such as buying and boycotting certain products.

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Introduction

Moderated Regression Analysis (MRA) -a particular specification of Multiple Linear Regression analysis- has been widely used (particularly in marketing research) for testing models that involve the presence of a variable that influences the impact of an independent variable on a dependent variable (Irwin and McClelland, 2001; Sethi et al., 2001; Keller, Lipkus and Rimer, 2003). However measurement error makes estimates of regression coefficients in MRA inconsistent and biased. Biased estimates -actually attenuated estimates- limit the use of the technique to purely predictive purposes. This bias is especially relevant for interaction effects that are usually of low magnitude (second order effects) and may easily go undetected if attenuated. Additionally, the estimated standard errors of regression coefficients are also biased; thus no coherent inferences about population parameters or relationships among variables can be made.

The use of Structural Equations Models (SEM) for correcting for measurement error has been proposed in the management literature, mainly by researchers in marketing (Ashok et al., 2002; Bagozzi and Yi, 1989; Homer, 1990; Martin and Stewart, 2001). However, SEMs have been proposed only rarely for estimating interaction effects (Ping, 1995).

In 1984 Kenny and Judd proposed a possible specification for modeling interaction effects with Structural Equation Models, known nowadays as the Moderate Structural Equation Model (MSEM). Kenny and Judd´s approach requires each latent variable to relate to at least two indicators and implies the formation of multiple indicators based on the products of the observed variables. These products are then used as indicators of the latent interaction.

Different alternatives have been proposed for developing Kenny and Judd's approach. It is not the aim of this paper to provide a comprehensive presentation of the various procedures currently available for testing interaction effects with MSEM (see for this purpose Li et al., 1998; Schumacker and Marcoulides, 1998, or Cortina et al., 2002), they are only conceptually clustered below as:

– Two-step approaches (Coenders et al., 2003; Jöreskog, 2000; Mathieu et al., 1992; Ping, 1995; Ping, 1996; Schumacker, 2002). In a first run, based on the main effects' indicators, they estimate certain parameter values of the measurement model or alternatively factor scores that are later used for the interaction factor in the second run.
– One-step two-stage least squares approach (Bollen and Paxton, 1998). It has the disadvantage of using a limited information estimator, which leads to poorer estimates (Moulder and Algina, 2002).
– One-step maximum likelihood approaches (Jaccard and Wan's, 1995; Jöreskog and Yang, 1996). Both approaches estimate the complete model with the main effects, interaction term, structural and measurement parts in one step. Both require specification of non-linear constraints, most of which are heavily based on the normality of main effects' indicators. Jöreskog and Yang use only one product indicator of the latent interaction and a mean and covariance structure, while Jaccard and Wan use multiple product indicators and a covariance structure. Due to the use of only one indicator, the Jöreskog and Yang approach shares typical features of limited information approaches if the indicators are not congeneric or the sample size is small, the results changing depending on which indicator is chosen (Saris, Batista-Foguet, and Coenders submitted to JMR).

Kenny and Judd's seminal article was published many years ago and MSEM applications have become rare in the marketing and management literature. As far as we know, only a methodologically-oriented article (Ping, 1995) has appeared in the field. As already mentioned, Moderate Regression is mostly used instead of MSEM, despite the former's obvious weakness in not taking measurement errors into account. This indicates that cumbersome approaches (complex non linear constraints) have deterred practitioners from using MSEM. This is hardly surprising, given that the approach requires a great deal of expertise in fitting SEM. Furthermore, not all SEM software can handle such constraints. Moreover, probably due to the influence of the Moderate Regression approach, every MSEM application (except Batista-Foguet et al., in press) is restricted to a single equation model, limiting SEM to only estimating direct effects.

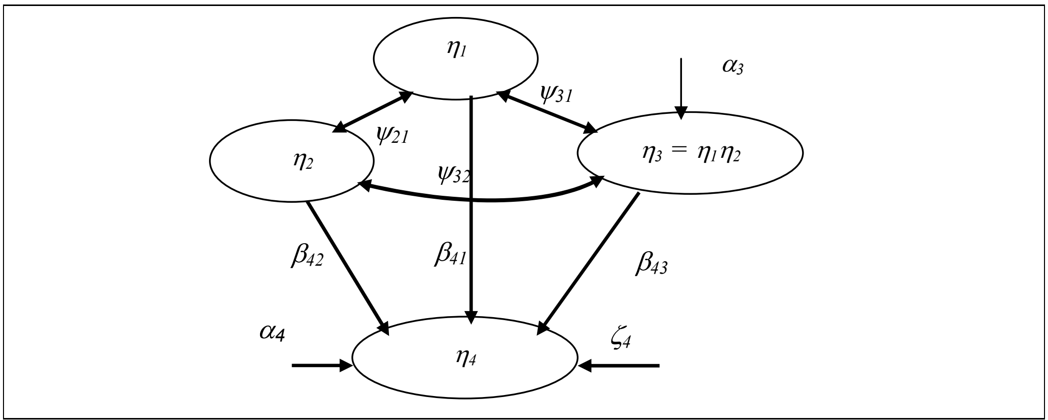
Since the two-step and two stage least squares approaches force the user to leave the conventional framework of structural equation modeling and make it difficult to compute either most of the diagnosis indexes commonly used in SEM or correct standard errors, this paper focuses on the maximum likelihood one-step strategy. This paper has two main objectives: First, the extension of the single equation approach to a simultaneous structural equation system in which the main effect and interaction terms may not be exogenous and in which variables may be related through a multitude of direct and indirect effects; Second, the paper critically addresses the problem of modeling and testing interaction hypotheses, contributing to the discussion among methodologists on SEM (Jöreskog and Yang, 1996; Ping, 1995; Ping, 1996; Jaccard and Wan, 1996; Li et al., 1998; Jöreskog, 1998; Algina and Moulder, 2001; Schumacker, 2002; Moulder and Algina, 2002; Cortina et al., 2002) by combining aspects of the different strategies that tend to make the approach simpler and relying on fewer assumptions. Our proposal shows that, when considering only two indicators for the latent interaction, constraints are unnecessary. So, we try to smooth the MSEM approach for practitioners and at the same time allow them to cope with a wider variety of situations by:

- Simplifying Jaccard and Wan's (1995) and Jöreskog and Yang's (1996) approaches by avoiding the need to use a mean structure and reducing or even eliminating the need for non-linear constraints.
- Using multiple (at least two) indicators of the latent interaction like Jaccard and Wan (1995), thus moving to a truly full information approach. Only non-overlapping pairs of indicators are used to compute the product thus leading to a simpler specification without correlated errors.
- Extending the applicability of the method to variables that are not normally distributed, by avoiding the non-linear constraints that assume normality.
- Generalizing the single equation structural model towards a simultaneous structural equation system. This would allow the researcher to have the interaction anywhere in the model, making it possible to estimate direct, indirect and total effects of the latent variables involved, thus exploiting the full strength of SEM.

The single equation model with one or two indicators

Before we present our proposals for extending and simplifying the approach, we start with the standard single equation model (Fig. 1) that has been specified for MSEM and presented in Equation 1:

Figure 1: Single equation SEM for modeling interaction effects



$$\eta_4=\alpha_4 +\beta_{41}\eta_1+ \beta_{42}\eta_2 + \beta_{43}\eta_3 + \zeta_4 \text{, where } \eta_3=\eta_1\bullet\eta_2 \qquad (1)$$

where β_{kl} stands for the regression coefficient of η_k on η_l and ζ_4 is the disturbance term, which - as usual - is assumed to be independent of η_1 and η_2. In this approach all three variables η_1, η_2 and η_3 are exogenous variables with free variances ψ_{kk} and covariances ψ_{kl}. Even if η_1 and η_2 are centered η_3 is not. So, other parameter of the model not shown on the equation are $E(\eta_3)=\alpha_3$. Furthermore we have $Var(\zeta_4)=\psi_{44}$.

It is assumed, without loss of generality, that the exogenous latent variables and the endogenous latent variables have two indicators. This is a key point in the paper since we will show that using two non-overlapping indicators does away with the need for the constraints required for identification purposes as discussed by Jaccard and Wan (1995) or by Jöreskog and Yang (1996).

In contrast with Jöreskog and Yang (1996), we specify a two-non-overlapping-interaction-indicator measurement model. The single indicator model is obtained by omitting the equation for y_6.

$$
\begin{aligned}
&y_1=\tau_1 + \eta_1 + \varepsilon_1 \\
&y_2=\tau_2 +\lambda_{21}\, \eta_1+ \varepsilon_2 \\
&y_3=\tau_3 + \eta_2 + \varepsilon_3 \\
&y_4=\tau_4 +\lambda_{42}\, \eta_2+\varepsilon_4 \\
&y_5=\tau_5+ \lambda_{51}\eta_1+\lambda_{52}\eta_2+\lambda_{53}\eta_3 + \varepsilon_5 \\
&y_6=\tau_6 +\lambda_{61}\, \eta_1+\lambda_{62}\eta_2+\lambda_{63}\, \eta_3 + \varepsilon_6 \\
&y_7=\tau_7 + \lambda_{74}\, \eta_4 + \varepsilon_7 \\
&y_8=\tau_8 + \lambda_{84}\, \eta_4 + \varepsilon_8
\end{aligned} \tag{2}
$$

where $y_5=y_1y_3$ and $y_6=y_2y_4$ and are thus independent non-overlapping measures (another possibility would be using y_1y_4 and y_2y_3). Without any loss of generality, the scale of the latent variables is fixed by constraining two loadings to 1.

$$\lambda_{11}=\lambda_{32}=1 \tag{3}$$

Additional parameters of the measurement part are Var$(\varepsilon_j)=\theta_{jj}$. The specification is completed with the assumptions that η_1, η_2, and ε_1 to ε_8 have zero expectation. Additionally, ε_1 to ε_8 are assumed to be mutually independent (not only uncorrelated) and independent of η_1, η_2, and ζ_4.

These assumptions allow us to analyze the expectation, variance and covariance of the product indicator, as well as to derive non-linear constraints, relating their associated parameters.

The product indicators, y_5, y_6, can be decomposed as:

$$y_5=y_1y_3=(\tau_1+\eta_1+\varepsilon_1)(\tau_3+\eta_2+\varepsilon_3)=\tau_1\tau_3+\tau_3\,\eta_1+\tau_1\eta_2+\eta_2\eta_1+\varepsilon_5 \tag{4}$$

$$y_6=y_2y_4=(\tau_2+\lambda_{21}\eta_1+\varepsilon_2)(\tau_4+\lambda_{42}\eta_2+\varepsilon_4)=\tau_2\tau_4+\tau_4\lambda_{21}\eta_1+\tau_2\lambda_{42}\eta_2+\lambda_{21}\lambda_{42}\eta_2\eta_1+\varepsilon_6 \tag{5}$$

where:

$$\varepsilon_5=\tau_3\varepsilon_1+{}_1\varepsilon_3+\eta_1\varepsilon_3+\eta_2\varepsilon_1+\varepsilon_1\varepsilon_3 \qquad (6)$$
$$\varepsilon_6=\tau_4\varepsilon_2+\tau_2\varepsilon_4+\lambda_{21}\eta_1\varepsilon_4+\lambda_{42}\eta_2\varepsilon_2+\varepsilon_2\varepsilon_4$$

The following constraints can be derived from the expressions of y_5 and $y_{6,}$ in Equations 2 to 6:

$$\begin{aligned}\tau_5&=\tau_1\tau_3\\ \lambda_{51}&=\tau_3\\ \lambda_{52}&=\tau_1\\ \lambda_{53}&=1\end{aligned} \qquad (7)$$

$$\begin{aligned}\tau_6&=\tau_2\tau_4\\ \lambda_{61}&=\tau_4\lambda_{21}\\ \lambda_{62}&=\tau_2\lambda_{42}\\ \lambda_{63}&=\lambda_{21}\lambda_{42}\end{aligned} \qquad (8)$$

The measurement error variances and covariances can also be derived, which involves the following constraints:

$$\theta_{51}=\mathrm{E}(\varepsilon_5\varepsilon_1)=\tau_3\,\mathrm{E}(\varepsilon_1\varepsilon_1)=\tau_3\,\theta_{11} \qquad (9)$$
$$\theta_{53}=\mathrm{E}(\varepsilon_5\varepsilon_3)=\tau_1\mathrm{E}(\varepsilon_3\varepsilon_3)=\tau_1\,\theta_{33}$$

$$\theta_{62}=\mathrm{E}(\varepsilon_6\varepsilon_2)=\tau_4\,\mathrm{E}(\varepsilon_2\varepsilon_2)=\tau_4\,\theta_{22} \qquad (10)$$
$$\theta_{64}=\mathrm{E}(\varepsilon_6\varepsilon_4)=\tau_2\mathrm{E}(\varepsilon_4\varepsilon_4)=\tau_2\,\theta_{44}$$

while

$$\theta_{65}=\mathrm{E}[(\tau_4\varepsilon_2+\tau_2\varepsilon_4+\lambda_{21}\eta_1\varepsilon_4+\lambda_{42}\eta_2\varepsilon_2+\varepsilon_2\varepsilon_4)(\tau_3\varepsilon_1+\tau_1\varepsilon_3+\eta_1\varepsilon_3+\eta_2\varepsilon_1+\varepsilon_1\varepsilon_3)]=0 \qquad (11)$$

as even overlapping pairs of terms in ε_5 and ε_6 are multiplying independent centered variables. It can also be shown that Cov$(\varepsilon_6\eta_1)$=Cov$(\varepsilon_6\eta_2)$= Cov$(\varepsilon_5\eta_1)$=Cov$(\varepsilon_5\eta_2)=0$.

The error variances can also be expressed as functions of other parameters, as:

$$\theta_{55}=\mathrm{Var}(\varepsilon_5)=\mathrm{Var}(\tau_3\varepsilon_1+\tau_1\varepsilon_3+\eta_1\varepsilon_3+\eta_2\varepsilon_1+\varepsilon_1\varepsilon_3)= \qquad (12)$$

$$=\tau_3^2\,\mathrm{Var}(\varepsilon_1)+\tau_1^2\,\mathrm{Var}(\varepsilon_3)+\mathrm{Var}(\eta_1)\mathrm{Var}(\varepsilon_3)+\mathrm{Var}(\eta_2)\mathrm{Var}(\varepsilon_1)$$
$$+\mathrm{Var}(\varepsilon_1)\mathrm{Var}(\varepsilon_3)=\tau_3^2\,\theta_{11}+\tau_1^2\theta_{33}+\psi_{11}\theta_{33}+\psi_{22}\theta_{11}+\theta_{11}\theta_{33}$$

$$\theta_{66}=\mathrm{Var}(\varepsilon_6)=\mathrm{Var}(\tau_4\varepsilon_2+\tau_2\varepsilon_4+\lambda_{21}\eta_1\varepsilon_4+\lambda_{42}\eta_2\,\varepsilon_2+\varepsilon_2\varepsilon_4)=$$
$$\tau_4^2\,\theta_{22}+\tau_2^2\theta_{44}+\lambda^2_{21}\psi_{11}\theta_{44}+\lambda^2_{42}\psi_{22}\theta_{22}+\theta_{22}\theta_{44} \qquad (13)$$

as under independence all covariances among any possible pairs of terms composing ε_5 or ε_6 are zero, even if they overlap.

The single indicator case omits y_6 and as a consequence the constraints in Equations 8, 10, 11 and 13. The remaining constraints that follow are common for both the single and the two-indicator cases.

The expectation of the interaction latent variable will be:

$$\alpha_3=\mathrm{E}(\eta_3)=\mathrm{E}(\eta_1\eta_2)=\mathrm{Cov}(\eta_1\eta_2)=\psi_{21} \qquad (14)$$

Other constraints are possible if the normality assumption is made besides the independence assumption (Jaccard and Wan 1995; Anderson 1984):

$$\psi_{33}=\mathrm{Var}(\eta_3)=\mathrm{Var}(\eta_1\cdot\eta_2)=\mathrm{Var}(\eta_1)\mathrm{Var}(\eta_2)+\mathrm{Cov}^2(\eta_1\eta_2)=\psi_{11}\,\psi_{22}+\psi^2_{21} \qquad (15)$$

$$\mathrm{Cov}(\eta_3\eta_2)=\psi_{32}=0;\ \mathrm{Cov}(\eta_3\eta_1)=\psi_{31}=0 \qquad (16)$$

Jöreskog and Yang suggested using only one indicator and constraints 7,9,12,14,15 and 16. Estimating the model with all these non-linear constraints requires a considerable technical capability of the researcher and it is even not possible to estimate this model with programs which do not include non-linear constraints. This is certainly one of the reasons why this approach is not frequently used in research. Fortunately, we will show in the next section that the estimation can be done with many fewer or even no restrictions at all.

Specification with Minimal constraints

Jaccard and Wan (1995) or Cortina et al (2002) suggested ignoring the mean structure. This involves centering the y_1 to y_4 indicators prior to computing the product indicators, and using only the covariance matrix as input for the model estimation (that is, recentering the interaction variables once they have been computed). Although the introduction of the mean structure can result in a gain of several degrees of freedom, no improvements in terms of bias or standard errors have been reported in a large simulation study (Moulder and Algina, 2002). Further advantages of using centered indicators and ignoring the mean structure are:

- Algina and Moulder (2001) report frequent non-convergence problems when the main effect indicators are not centered.
- With the LISREL program, a complicated parameterization (see Jöreskog and Yang, 2001 and appendix 2) must be used for robust estimation with mean structures, which requires even more expertise from the modeler.
- Centering variables prior to computing the product, minimizes the relationships between the variables and the product computed from them, which reduce collinearity (See Li et al., 1998, and the Appendix in Irwin and McClelland, 2001). The fact that the interaction indicators now load only on the interaction latent variable and not on the main effect latent variables also reduces collinearity between the main effect and interaction indicators.
- The number and complexity of non-linear constraints is greatly reduced.

If the variables are centered, then all τ and α parameters are zero and from (7) to (11) it follows that $\lambda_{51}=\lambda_{52}=\lambda_{61}=\lambda_{62}=0$ and $\theta_{51}=\theta_{53}=\theta_{62}=\theta_{64}=0$. Thus many restrictions are no longer needed and only the following constraints remain:

$$\lambda_{53}=1 \tag{7b}$$

$$\lambda_{63}=\lambda_{21}\lambda_{42} \tag{8b}$$

$$\theta_{55}=\psi_{11}\theta_{33}+\psi_{22}\theta_{11}+\theta_{11}\theta_{33} \tag{12b}$$

$$\theta_{66}=\lambda^2_{21}\psi_{11}\theta_{44}+\lambda^2_{42}\psi_{22}\theta_{22}+\theta_{22}\theta_{44} \tag{13b}$$

$$\psi_{33} = \psi_{11}\ \psi_{22} + \psi^2_{21} \tag{15b}$$

$$\psi_{32}=\psi_{31}=0 \tag{16b}$$

Constraints 15, 16, 15b and 16b make use of a result regarding the variance of the product of two normal variables. If the original measures are severely non-normal, then the variance of the product variable can be very different from the value implied by their development, and the interaction model may perform poorly" (Rigdon et al., 1998). Our own simulations using several distributions for η_1 and η_2 showed substantial departures of ψ_{33} from $\psi_{11}\ \psi_{22} + \psi^2_{21}$ and of ψ_{31} and ψ_{32} from zero. Under a correlation of .7 between η_1 and η_2, relatively small skewness and kurtosis (-1 and +1) resulted in deviations between ψ_{33} and $\psi_{11}\ \psi_{22} + \psi^2_{21}$ in the 30%-40% range. In the case of skewness around 1, the correlation between η_3 and η_2 or η_1 was around 0.4.

Fortunately, these constraints are not necessary for identification, even in the single indicator case. Jöreskog (personal communication, February 14, 1995) indicated that researchers can deal with non-normality merely by relaxing some of the constraints that are part of the Kenny-Judd interaction model. Thus, the cost of non-normality here may be no more than a loss of parsimony (Rigdon et al., 1998). However, these constraints are customarily introduced by practitioners without performing any normality test, likely because of following to the letter the influential work of Jöreskog and Yang (1996). So we suggest avoiding these constraints.

Jaccard and Wan (1995) suggested using all constraints 7b to 16b. However, for identification purposes, only 7b is needed. Additionally, they introduced all other possible pairs of product indicators *(y_1y_4* and *y_2y_3)* even if not needed either for the model identification. The error covariances between overlapping pairs *(y_1y_4-y_1y_3; y_1y_4-y_2y_4; y_2y_4-y_2y_3; y_1y_3-y_2y_3)* were left free, though it would have been possible to constrain them to appropriate non-linear functions of model parameters. For instance, the covariance between y_1y_4 and y_1y_3 is $\lambda_{42}\psi_{22}\theta_{11}$.

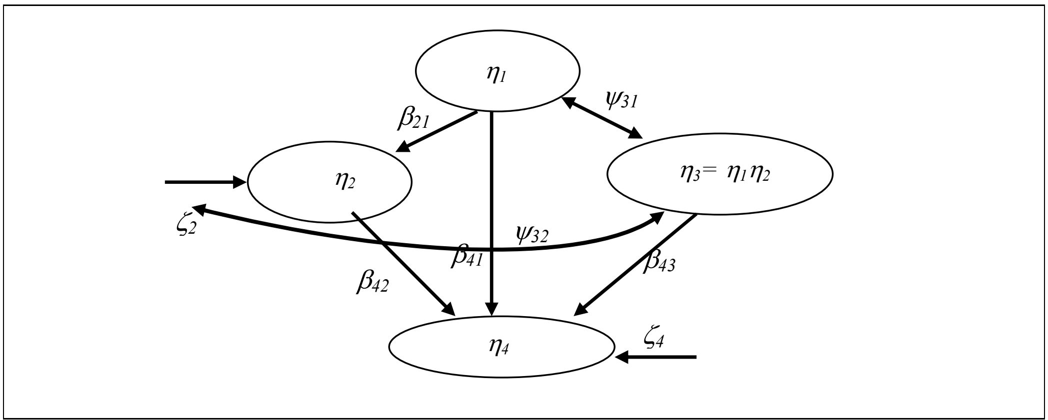
In this paper we follow Jaccard and Wan in using centered indicators (as mentioned, this makes things a great deal simpler and workable). However, as regards constraints and selection of indicators we suggest departing from Jaccard and Wan and Jöreskog and Yang in the following respects:

- In the single indicator case, only constraint 12b is needed for identification and will be used. Constraints 15b and 16b, which rely on normality, are thus omitted.
- In the multiple-indicator case, only the linear constraint 7b is needed for identification, although the very simple non-linear constraint 8b can also be used if software permits. 12b and 13b do not require normality and may also be used by expert modelers but are not necessary for identification.
- We also suggest dropping overlapping pairs of indicators as suggested by Bollen and Paxton (1998) and Schumacker (2002) as they only unnecessary complexity in exchanging information that is actually redundant.

Extension to a simultaneous structural equation system

MSEM have so far usually been modeled with one equation, where the regressors that interact are exogenous latent variables (Schumacker and Marcoulides 1998; Li et al., 1998; Schumacker, 2002; Moulder and Algina, 2002; Cortina et al., 2002). This single equation formulation, discussed in the SEM literature, only makes it possible to estimate direct and interaction effects. In order to also estimate for instance an indirect effect in the same model, a simultaneous structural equation system must be specified for including direct, indirect and moderating effects in the same model. We show a very simple example in Figure 2.

Figure 2: SEM including direct, indirect and interaction effects simultaneously



The structural part of the model would include equation 1b (without the α term) and an additional equation for the relationship among the up to now exogenous variables, η_1 and η_2.

$$\eta_2 = \beta_{21}\eta_1 + \zeta_2 \tag{17}$$

Only η_1 and $\eta_3 = \eta_1\eta_2$ remain exogenous. Additional parameters of the structural part are: ψ_{11}, ψ_{22}, ψ_{33}, ψ_{32} and ψ_{31}. It must be taken into account that ψ_{32} is not the covariance between η_2 and η_3 but between ζ_2 and η_3 . Besides, now $\psi_{21}=0$.

The main difference with the previous specification is that not all the variances and covariances of η_1 and η_2 are model parameters but rather functions of model parameters that can be derived from path analysis or from variance and covariance algebra:

$$\mathrm{Var}(\eta_2) = \psi_{22} + \beta^2_{21}\psi_{11} \tag{18}$$

$$\mathrm{Cov}(\eta_1, \eta_2) = \beta_{21}\psi_{11} \tag{19}$$

The assumptions are the same as before with the addition that ζ_2 is uncorrelated with η_1.

Substituting in Equations 12b to 16b the new expression for Var(η_2) and Cov(η_1, η_2) yields:

$$\theta_{55} = \psi_{11}\theta_{33} + (\beta_{21}{}^2\ \psi_{11} + \psi_{22})\theta_{11} + \theta_{11}\theta_{33} \tag{12c}$$

$$\theta_{66} = \lambda^2{}_{21}\psi_{11}\theta_{44} + (\beta_{21}{}^2\ \psi_{11} + \psi_{22})\lambda_{42}\theta_{22} + \theta_{22}\theta_{44} \tag{13c}$$

$$\psi_{33} = \psi_{11}\ (\psi_{22} + \beta^2{}_{21}\ \psi_{11}) + (\beta_{21}\ \psi_{11})^2 \tag{15c}$$

$$\mathrm{Cov}(\eta_3\eta_2) = \psi_{32} + \beta_{21}\psi_{31} = 0;\ \mathrm{Cov}(\eta_3\eta_1) = \psi_{31} = 0 \tag{16c}$$

The model in this section also estimates the relationship between η_1 and η_2 and thus both a direct and an indirect effect from η_1 to η_4. The joint interpretation of indirect and interaction effects is discussed in Batista-Foguet et al. (in press), who basically fitted the same model with the single indicator approach and constraints 12c and 15c. Their analysis thus required normality and included highly complex constraints for this relatively simple model. This complexity would grow to unbearable levels even for moderately sophisticated models. Just imagine a model including exogenous variables affecting η_1 and η_2.

As was mentioned, for the single indicator approach, constraint (12c) is the only to be needed. It does not require normality, but it has increased its complexity due to the fact that Var(η_2) is a function of model parameters. For the two-indicator approach, none of the constraints is needed, though constraint 8b has not increased its complexity and may be introduced if so wished. Constraints 15c and 16c require normality and are never required or advised.

Although we illustrated this issue for a simple model, this result holds true for SEM models with interaction term anywhere in the model. This means that these interaction effects can be estimated with a minimal burden of extra restrictions.

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An illustration

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Data and Measurement Instruments

Our approach is illustrated using one example that has been discussed also by Saris, Batista-Foguet, and Coenders (submitted to JMR). This example is based on data from the British pilot study of the European Social Survey (2002). In that study, a variable has been measured indicating "environmental friendly customer behavior" like buying and boycotting certain products *(η_4)*. Furthermore, measures are available for "value orientations" with respect to the environment *(η_1)* and for the possibilities of influencing events, "political influence variable" *(η_2)*. See in Appendix 1 for the two items-indicators corresponding to each of these variables. We expect that the value orientation will only affect behavior if people believe they can influence the situation. So we expect an interaction effect of these two variables which is represented by η_3.

Sample size was 429 and EM imputation of missing values was used. The main effect indicators are non-normally distributed. The maximum absolute skewness was .75 for y_4 and the maximum absolute kurtosis was -.92 for y_1. As a consequence, we are reluctant to include constraints that assume normality in our specification, and we will show the advantages of our proposal by comparing the following models:

(a) Single indicator with mean structure only with constraints not requiring normality: 7, 9, 12c adapted to include τ parameters, 14 adapted to include indirect effects;
(b) Two interaction indicators plus constraints: 8b, 12c, 13c;
(c) Two interaction indicators plus constraint: 8b;
(d) Two interaction indicators without constraints.

Estimation method

Normality of the interaction indicators is not assumed by any of the analyses and would be difficult to fulfill, as the product of two normal variables is in general not normally distributed. This calls for robust standard errors and test statistics like the ones described by Satorra and Bentler (1988, 1994). The LISREL8.5 program (Du Toit and Du Toit 2001) includes these robust statistics and allows the researcher to introduce non-linear constraints and is thus appropriate for the estimation of this model and will be used in this paper. Maximum likelihood estimation is used throughout.

Results

In this section the models presented before are fitted to the data of the example using either one (analysis a in Table 1) or two indicators (analyses b to d in Table 1) for the latent interaction, and using alternative sets of constraints, except those constraints requiring normality. Some constraints are correct under milder assumptions but grow in complexity with the model (Equations 12c, 13c as in analyses a and b), other constraints are always strictly observed and kept simple (Equation 8b as in analysis c).

The inclusion of additional constraints is expected to reduce standard errors, though some are unworkable for large models. As already mentioned, the inclusion of constraints requiring normality is likely to affect the consistency of estimates if normality does not hold. Thus, constraints in Equations 8b, 12c, and 13c should lead to the best results in terms of efficiency but certainly not in terms of simplicity. Thus, we expect the two-indicator case with constraints 8b to be a sensible trade off between the two requirements (analysis c).

Table 1: Estimates and standard errors of key parameters and goodness of fit indexes under different assumption

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Parameter/Index	(a)	(b)	(c)	(d)
β_{43}	-.268	-.198	-.198	-.184
s.e β_{43}	.070	.054	.056	.066
β_{42}	.500	.473	.474	.477
s.e β_{42}	.076	.069	.069	.069
β_{41}	.168	.158	.158	.158
s.e β_{41}	.039	.033	.033	.033
β_{21}	.029	.010	.009	.008
s.e. β_{21}	.040	.037	.037	.036
Satorra-Bentler χ^2	11.76	15.45	14.51	14.04
d.f.	10	18	16	15
p-value	.301	.631	.561	.523
RMSEA	.021	.000	.017	.000
90%CI RMSEA	.00; .058	.00; .036	.00; .041	.00; .043
SRMR	.005	.019	.019	.020

(a) Single indicator with mean structure plus constraints: 7, 9, 12c, 14.
(b) Two interaction indicators plus constraints: 8b, 12c, 13c.
(c) Two interaction indicators plus constraint: 8b.
(d) Two interaction indicators without constraints.

As shown by the Satorra-Bentler Scaled χ^2, RMSEA and SRMR values, the fit of all models was good enough to proceed to the comparison of their results.

A first look at the table tells us that, in general, differences are not very large so that the simplest approaches, like analyses c and d, all using two indicators and simple or no constraints, can be used instead of the very sophisticated (a) approach of Jöreskog and Yang (1996).

Going into detail:

- The addition of constraints to a given model tends to reduce standard errors, but it does so by such a narrow margin that it is hardly worth the additional complexity. Actually, standard errors of analysis (c) are not generally larger than those from other analyses including complex constraints, and are even smaller than those of analysis (a) that includes the information contained in the means. The introduction of additional indicators generates degrees of freedom that seem to more than offset the omission of the means and of constraints.

- In the two indicator case, if no non-linear constraints are used, virtually all software programs handling SEM could be used. In Appendix 2 we also present the setup for analysis (d) with EQS 5 for Windows (Bentler and Wu, 1995), a program that does not include non-linear constraints. The price that has to be paid is a slight increase in standard errors with respect to analysis (c).
- The addition of a second interaction's indicator does alter the point estimate of β_{43}, in the way that can be expected when moving from a method which in some respect is one of limited information to one of full information. Differences can be even larger if the items are not exactly congeneric (Saris, Batista-Foguet and Coenders submitted to JMR) or if the sample is small, which makes us recommend use of multiple interaction indicators on a general basis.

Conclusions and recommendations

The paper provides a more simplified and parsimonious specification and an extension of the usual approaches for modeling interactions using SEM.

The main idea underlying the simplified approach stems from the fact that there are still very few MSEM users in comparison with those modeling interaction effects. Researchers keep using MRA instead because MSEM simply requires too much methodological expertise.

Since Kenny and Judd (1984), all other approaches have involved two step methods with unclear theoretical statistical properties or one step methods with very complicated non-linear constraints, some of which required normality which, according to our simulations, is a very unwise assumption.

This paper has shown that while the single indicator model by Jöreskog and Yang (1996) requires the introduction of non-linear constraints for model identification purposes, when using at least 2 indicators for the latent interaction and when omitting the mean structure, no non-linear constrains are need or advisable.

The main idea underlying the extension of the approaches used so far is that by restricting MSEM to one equation model they are actually diminishing the potentiality of SEM. So, MSEM should also being able to cope with relatively complex models, including indirect effects as well as the direct ones. The elimination of the need for complex constraints, makes the approach much more workable, independent of the complexity of the model and the fulfillment of the assumptions (not requiring normality) while allowing applied researchers to fit these models easily even with standard software (without non-linear constraints),

So, a very practical recommendation is to use two or more indicators for the latent interaction and:

- For a simple model or even a model with a single equation, constraints 8b, 12b-13b are relatively simple, do not require normality and can be used if one wishes to attain a marginal increase in efficiency, but they are not needed for identification. This is illustrated in analysis (b).
- For a model with several equations, only constraint 8b remains simple and can be used though it is not required for identification, as we did in analyses (c).
- For a researcher whose software cannot handle non-linear constraints, even constraint 8b can be dropped. This has been our approach for analysis (d).

If the main effects have more than two indicators, it is also possible to form a larger number of non-overlapping pairs as indicators of the latent interaction, which will result in even smaller standard errors. As before, no constraints are needed.

As regards estimation, since the product indicator will not be normally distributed even if the main effect indicators are. Methods are needed that are robust to departures from normality (Yang and Jöreskog 2001). However, it must be noted that these methods do not correct the bias incurred if constraints 15b, 16b, 15c or 16c are introduced under non-normal main effect factors. This is so because robust methods in SEM only improve the correctness of standard errors and test statistics under violation of distributional assumptions, not the consistency of point estimates under introduction of wrong constraints (e.g. Satorra, 1990).

Appendix 1. The measurement of the different variables of Figure 1.

In the ESS pilot study, the following measures were available for the different variables of interest. **"Value orientations"** with respect to the environment" were measured by items of Schwartz's (1997) value scale. In the pretest of the European Social Survey, the original item -containing two parts- was split into two separate items: an importance statement and a norm. Saris, Batista-Foguet, and Coenders (submitted to JMR) have shown that these items are congeneric which means that they measure only the same variable (η_1).

The two items were presented to the respondents in the following form:

How much like you is this person?

y_1: Values1. *Looking after the environment is important to him/her.*
y_2: Values2. *He/she strongly believes that people should care for nature.*

1 very much like me, **2** like me, **3** somewhat like me, **4** a little like me, **5** not like me, **6** not like me at all

Two question was asked to measure "**political influence**" *(η_2)*: Also these questions are congeneric as shown by Saris, Batista-Foguet, and Coenders (submitted to JMR)

How far do you agree or disagree with each of the following statements?

y_3: Control1. *I think I can take an active role in a group that is focused on political issues.*
y_4: Control2. *Exactly the same statement was repeated after a period of time (only the response scale was reversed).*

1 very much like me, **2** like me, **3** somewhat like me, **4** a little like me, **5** not like me, **6** not like me at all

The dependent variable is a **"environmental friendly customer behavior"** variable measuring purchasing and boycotting of products and other items for environmental and other political reasons (η_3). This measure has been asked in two different ways, the second at the end of the supplementary methodological questionnaire (after approximately 45 minutes of other questions):

y_7**: Buy1.** *How many of the four things on this card have you done during the last 12 months?*

1. *Deliberately bought certain products for political, ethical or environmental reasons*
2. *Boycotted certain products*
3. *Donated money*
4. *Raised funds*

y_8**: Buy2.** *During the last 12 months, have you done any of the following?*

1. *Deliberately bought certain products for political, ethical or environmental reasons*
2. *Boycotted certain products*
3. *Donated money*
4. *Raised funds*

1 Yes, **2** No

Buy1 automatically provides a score between 0 and 4. For Question **Buy2,** the number of "yes" answers were summed to get the total score.

Appendix 2: LISREL setups for Models (a) and (c); EQS setup for model (d)

```
LISREL8 setup.  Yang-Jöreskog  approach (Model a).
! AUGMENTED MOMENT MATRIX USED AS COVARIANCE MATRIX.
! LAST COLUMN OF LAMBDA CONTAINS TAU PARAMETERS,
! LAST COLUMN OF BETA CONTAINS ALPHA PARAMETERS.
DA NI=9 NO=429 MA=CM
LA
VALUES1   VALUES2  CONTROL1  CONTROL2   INTY1Y3   BUY1  BUY2  const
CM = 8origimp.cm
AC= 8origimp.ac
MO NY=8 NE=5 LY=FU,FI BE=FU,FI TE=SY,FR PS=SY
LE
values control interact buying CONSTANT
PA LY
0 0 0 0 1
1 0 0 0 1
0 0 0 0 1
0 1 0 0 1
1 1 0 0 1
0 0 0 0 1
0 0 0 1 1
0 0 0 0 0
PA TE
0
0 1
0 0 1
0 0 0 1
0 0 0 0 1
0 0 0 0 0 1
0 0 0 0 0 0 1
0 0 0 0 0 0 0 1
PA BE
0 0 0 0 0
1 0 0 0 0
0 0 0 0 1
1 1 1 0 0
0 0 0 0 0
PA PS
1
0 1
1 1 1
0 0 0 1
0 0 0 0 1
ST 1 LY(2,1) LY(4,2) LY(7,4)
VA 1 LY(1,1) LY(3,2) LY(5,3) LY(6,4) LY(8,5)
co ly(5,5)= ly(1,5)*ly(3,5)
co ly(5,1) = ly(3,5)
co ly(5,2) = ly(1,5)
co te(5,5) = ly(1,5)**2*te(3,3)+ly(3,5)**2*te(1,1)+ps(1,1)*te(3,3)+ps(2,2)*te(1,1)+be(2,1)**2*ps(1,1)*te(1,1)+te(3,3)*te(1,1)
co te(5,1) = ly(3,5)*te(1,1)
co te(5,3) = ly(1,5)*te(3,3)
co be(3,5) = be(2,1)*ps(1,1)
OU ML RS EF
```

```
LISREL8 setup. Model (c) with two interaction indicators. Simple constraint: 8b
DA NI=8 NO=429
 LA
 VALUES1   VALUES2   CONTROL1  CONTROL2   INTY1Y3   INTY2Y4   BUY1  BUY2
 CM fi=cov.cm re
 AC fi=ascov.ac re
 MO NY=8 NE=4 LY=FU,FI TE=SY,FR PS=SY BE=FU,FI
 LE
 VALUES CONTROL INTERACT BUYING
 PA LY
 0 0 0 0
 1 0 0 0
 0 0 0 0
 0 1 0 0
 0 0 0 0
 0 0 1 0
 0 0 0 0
 0 0 0 1
 PA TE
 0
 0 1
 0 0 1
 0 0 0 1
 0 0 0 0 1
 0 0 0 0 0 1
 0 0 0 0 0 0 1
 0 0 0 0 0 0 0 1
 PA BE
 0 0 0 0
 1 0 0 0
 0 0 0 0
 1 1 1 0
 PA PS
 1
 0 1
 1 1 1
 0 0 0 1
 ST 1 LY(2,1) LY(4,2)
 VA 1 LY(1,1) LY(3,2) LY(5,3) LY(7,4)
 co LY(6,3) = LY(2,1)*LY(4,2)
 OU ML RS EF
```

```
/TITLE
  EQS Setup for Model (d) with two interaction indicators. No constraints at all.
/SPECIFICATIONS
  DATA='C:\EQS\SIXINTE1.ESS'; VARIABLES=  8; CASES=  429;
  METHODS=ML,ROBUST; MATRIX=RAW;
 /LABELS
  V1=values1; V2=values2; V3=control1; V4=control2; V5=INTy1y3; V6=INTy2y4; V7=buy1; V8=buy2;
 /EQUATIONS
   V1 =  + 1F1  + E1;
   V2 =  + *F1  + E2;
   V3 =  + 1F2  + E3;
   V4 =  + *F2  + E4;
   V5 =  + 1F3  + E5;
   V6 =  + *F3  + E6;
   V7 =  + 1F4  + E7;
   V8 =  + *F4  + E8;
   F4 =  + *F1  + *F2  + *F3  + D4;
   F2 =  + *F1  + D2;
 /VARIANCES
   F1 = *;    F3 = *;
   E1 = 0;   E2 = *;   E3 = *;    E4 = *;   E5 = *;    E6 = *;    E7 = *;    E8 = *;
   D4 = *;    D2 = *;
 /COVARIANCES
    D2 , F3 = *;
    F1 , F3 = *;
  /END
```

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