Imperfect Competition and Market Liquidity with a Supply Informed Trader

Prof. Dr. Ariadna Dumitrescu
Department of Financial Management and Control
ESADE Business School

February 2005

Abstract

We develop a model of insider trading where agents have private information either about liquidation value or about supply, and behave strategically to maximize their profits. The presence of a supply-informed trader in the market induces non-monotonicity of market indicators with respect to the variance of liquidation value. Moreover, the existence of private information about supply affects significantly market performance as it induces, among other effects, lower market liquidity. Finally, our model suggests another link between Kyle's (1985, 1989) and Glosten and Milgrom's (1985) models by allowing for strategic behavior of an informed dealer.

JEL Classification numbers: D82, G12, G14.

Keywords: Insider trading, Imperfect competition, Market liquidity.

Introduction

Agents engaged in trading activities might have access to different sources of information: information about fundamentals or information about supply. On the one hand, there are agents who acquire information about fundamentals, which are predictors of future prices. On the other, there are agents who, due to their position in the market, might have access to the order book and can therefore gather information about the supply side of the market. The existence of these two different types of information in the market might have some bearing on the inefficiencies that appear when agents are trading on private information concerning fundamentals. It has been shown that in an imperfect competitive setup, traders exploit their informational advantage by taking into account the effect the quantity they choose is expected to have on both the price and the strategy adopted by other traders. The strategic use of this private information is even more important when we consider different types of information. The aim of this paper is to analyze the process through which different types of information are transmitted to prices and the implications of strategic trading on the market performance in this new setup. In order to do so, we develop a model of insider trading in the context of an imperfectly competitive market - similar to Kyle (1989) - where agents have private information either about future prices or about supply. This distinction between value-informed traders and supply-informed traders is designed to capture the different types of information that influence the security prices at any given point in time.

In the Kyle-type models, an important assumption is the presence of noise. As explained by Grossman and Stiglitz (1980), noise is needed in the model to prevent prices from being fully revealing. They show that in a model in which agents are price takers and prices are fully revealed, no agent will be willing to acquire costly information. To overcome this difficulty, several ways of introducing noise were used: adding noise traders, considering uncertainty which has a dimension greater than that of price, or assuming that the aggregate endowment is imperfectly observed. We use this last approach by assuming a random supply. The presence of shocks in supply has a significant price impact. A supply shock leads to a change in prices and this makes investors revise their expectations. However, if the supply shock is observable by the supply-informed traders, these traders are making use of their informational advantage and therefore are willing to adjust their demand. Consequently, we assume that there is a supply-informed trader who receives a signal about supply. This approach was used before by Gennotte and Leland (1990) who consider a model where speculators possess private and diverse information.[1] They consider price taker speculators who gather information either about prices or about supply and show that these informational differences can cause financial markets to be relatively illiquid. Our model builds on the assumption by Gennotte and Leland (1990) concerning the existence of a random supply and supply-informed speculator but we consider an imperfect competition setup with both value-informed and supply-informed agents where the agents submit limit orders. In general, dealers have access to the order book and thus they can observe the order flow and collect information from multiple sources. We can therefore think of the supply-informed agent as being a dealer. As pointed out by Brown and Zhang (1997), despite the fact that dealers may be better informed than other traders, in a competitive market they cannot earn rents from the information on the order flow. This is due to the fact that price informed agents use their informational advantage to make gains at the dealers' expense. However, we will see that in our setup of an imperfect competitive market, dealers can aggregate the information from trading and use it to earn speculative profits. Thus, dealers can learn about the liquidation value of the asset from the orders placed by the price informed agents. The information revelation is increased significantly in our setup since the agents are placing limit orders and therefore, they condition their demands on prices and hence, infer part of the others' information.

In the rational expectations paradigm, traders understand that prices reveal the information they have when they choose the quantities to be traded. The link

[1] A similar assumption is that market makers have some information about the uninformed order flow and it can be found in Admati and Pfleiderer (1991) and Madhavan (1992). Palomino (2001) considers also a setup where the informed agents have information both about the liquidation value and the quantity traded by one of the noise traders.

between information and prices via trades provides an explicit mechanism for information transmission between traders. The existence of private information means that a trader may have incentives to act strategically in order to maximize his profits. Therefore, given his private information, a trader maximizes his conditional expected profits taking into account the effect of his trading on prices and taking as given the strategies other traders use to chose their demand schedules. As in the imperfect competition model of Kyle (1989), we further assume that all traders strategically choose the amounts they trade. Therefore, the supply-informed trader also chooses his demand, taking into account the effect of his trading on prices and revealing some information about the shock in supply to other market participants. As a result, in our model both the information about the value of the asset and about supply is revealed through the quantities to be traded.

In our model, we use the framework developed by Kyle (1985, 1989) which has become a standard for analyzing strategic noisy rational expectations markets. Kyle's (1985) model explains how a risk neutral informed trader exploits his informational advantage by behaving strategically and shows that the smoothing behavior of the trader leads to prices that have constant volatility as the time periods become shorter to approach a continuous auction. An important generalization of Kyle's model allows for multiple informed traders. Since the monopolist trader makes positive profits, it follows that other trade might be willing to acquire information. Foster and Viswanathan (1993) and Holden and Subrahmanyam (1992) explore this restriction of a single informed trader and point out the contrast between the case of a monopolist and the one of multiple traders. Thus, Foster and Viswanathan (1993) extend Kyle's model to many traders and a larger class of distributions but find that Kyle's finding that an informed trader can make positive profits no longer obtains under such circumstances. On the other hand, Holden and Subrahmanyam (1992) conclude that competition between informed traders leads to full revelation of information. A further extension is proposed by Caballé and Krishnan (1994). They study a multi-security market with risk neutral agents in a correlated setup and they generalize Kyle's (1985) finding that more noise leads to more aggressive trading. Moreover, an important result is that in their model, portfolio diversification arises due to the strategic behavior of the agents and not because of risk considerations. Kyle (1989), to which our work is closely related, proposes an imperfect competition model in which there are noise traders, price-informed traders and uninformed traders. He shows that a strategic trader acts as he trades against a residual supply curve. This implies lower quantities by comparison with the competitive rational expectations equilibrium and, consequently, equilibrium prices reveal less information than in the competitive case. As will be emphasized in this paper, the dual role of prices in aggregating information and clearing the market is even more important when we have different types of information.

In this paper we are interested to see how market performance is influenced by the strategic interaction between agents with different types of private information. Consequently, we analyze the effect these different types of information have on several market indicators: market liquidity, informativeness of prices, price volatility, expected volume and traders' expected profit. We perform comparative statics results for these market indicators and conclude that the homogeneity of the signals plays an important role in market performance. Most important, the presence of the supply-informed agent induces non-monotonicity of the market depth and other market indicators with respect to the variance of the liquidation value.

Our model suggests that allowing the supply-informed agent to behave strategically has an important role in market-making and in information aggregation. Indeed, we find that the supply-informed trader decreases market depth and increases the amount of information revealed in prices. Moreover, unlike in the perfectly competitive case, this trader also makes positive profits. An interesting implication of our model is that the presence of different types of information in the market decreases market liquidity. This result is similar to the one in the dual trading literature and this is not at all surprising. Despite initially possessing only one type of information, both value-informed and supply-informed traders end up trading on the two types of information, as the brokers-dealers do in the dual trading literature. However, unlike in this literature, in our model we also obtain other important implications with respect to market performance. Finally, notice that our model is related to Kyle's (1989) but produces results consistent with Glosten and Milgrom's

(1985), which shows that more information in the market leads to an increase in the bid-ask spread (i.e. a decrease in the market liquidity). As shown by Krishnan (1992) and Back and Baruch (2004), the two separate strands of literature (to which Kyle (1985) and Glosten and Milgrom (1985) belong, respectively) are in fact intertwined. The suggested link is an equivalence between the extensive forms in Krishnan (1992), and a convergence process of the equilibria in Glosten and Milgrom (1985) to the equilibrium in Kyle (1985) in Back and Baruch (2004). Our work suggests that the compatibility of the results produced by the two families of models may have a dimension other than the ones revealed in Krishnan (1992) and Back and Baruch (2004) by allowing for strategic behavior by informed dealer.

The remainder of this paper is organized as follows. Section 2 presents the model. We establish the information structure and define the imperfect competitive rational equilibrium expectations. Section 3 characterizes the equilibrium. We find a unique linear imperfect competitive rational expectations price function together with agents' demand functions in equilibrium. Section 4 proceeds with the calculation of some market indicators: volatility of prices, informativeness of prices and expected profits. Section 5 contains the characterization of the equilibrium in the case there is no supply-informed trader and then Section 6 compares the market indicators of this economy with the one of the economy with a supply-informed agent. Finally, Section 7 summarizes the results. All the proofs appear in the appendix.

The Model

The framework is similar to the one in Kyle (1989). However, we assume risk neutrality, absence of uniformed traders and random supply with an observable component for one trader - the supply-informed trader. As already pointed out by Kyle (1989), the assumption of the existence of uninformed traders does not change the analysis, but their presence leads to an increase in market depth. We model the random supply that keeps traders from perfectly inferring the aggregate information from prices in a similar manner to the one in Gennotte and Leland (1990), but we assume here that there is only one supply-informed trader. Made for simplicity, the assumption is in line with the result obtained by Ellis et al. (2001). They show that in general, one dealer tends to dominate trading of a stock (executing a little more than half of the day's volume). They also answer the question as to who is the dominant dealer. Depending on the time elapsed from the offer day, the dominant dealer might be the underwriter, a wholesaler or a generic market maker. In what follows, we make the following assumptions:

A.1 There is a single security in the market that trades at market clearing price $\widetilde{p}$ and yields an exogenous liquidation value $\widetilde{v}$, which has a normal distribution with mean $\overline{v}$ and variance σ_v^2.

A.2 There are N value-informed traders, indexed $n = 1, ..., N$ and a supply-informed trader. The price informed trader n observes a private signal $\widetilde{i_n} = \widetilde{v} + \widetilde{e_n}$. We assume that e_n is distributed $N(0, \sigma_e^2)$ for all $n = 1, ..., N$. We suppose that for any $j \neq n$ $\widetilde{e_j}$ and $\widetilde{e_n}$ are uncorrelated and moreover, they are uncorrelated with all the other random variables in the model. The supply-informed trader observes a private signal S which is normal distributed with mean 0 and variance $\sigma_S^2 > 0$.

A.3 The net supply $\widetilde{m}$ consists of a fixed amount $\overline{m}$ and a random supply $\widetilde{S}$ distributed $N(0, \sigma_S^2)$. This liquidity shock $\widetilde{S}$ is observed only by the supply-informed trader.

A.4 Agents are risk neutral and behave strategically taking into account the effect of their trading on prices.

As in Kyle (1989), the n^{th} value-informed trader has a strategy X_n which is a mapping from $\mathbb{R}^2$ (the cartesian product of the set of asset prices and the set of his signals) to $\mathbb{R}$ (the set of shares he desires to trade), $X_n(\cdot, \cdot) : \mathbb{R}^2 \longrightarrow \mathbb{R}$. After observing his signal i_n, each value-informed trader submits a demand schedule (or generalized limit order) $X_n\left(\cdot, \widetilde{i_n}\right)$, which depends upon his signal. Similarly, the supply-informed trader has a strategy Y, which is a mapping from $\mathbb{R}^2$ (the cartesian product of the set of asset prices and the set of his signals) to $\mathbb{R}$ (the set of shares he wants to trade), $Y(\cdot, \cdot) : \mathbb{R}^2 \longrightarrow \mathbb{R}$. After observing the signal S, the supply-informed trader chooses a demand schedule $Y(\cdot, S)$, which depends upon that signal. Notice that since $\overline{m}$ is known by everyone, this implies that the supply-informed agent actually knows $\widetilde{m}$. Given a market clearing price p, the quantities traded by value-informed traders and supply-informed trader can be written $x_n = X_n(p, i_n)$, $n = 1, ..., N$ and $y = Y(p, S)$. In the above notations, a tilde distinguishes a random variable from its realization. Thus, x_n denotes a particular realization of $\widetilde{x}_n$. The assumption that the value-informed and the supply-informed agents submit limit orders for execution against existing limit orders submitted by the other market participants turns out to be very important (for a detailed discussion see Kyle (1989)). In this context both the value-informed and the supply-informed agents provide liquidity and therefore, play a market-making role.

The price of the asset is set such that the market clears. The traders submit their demand schedules to an auctioneer who aggregates all the schedules submitted, calculates the market clearing price and allocates quantities to satisfy traders' demand. Thus, the market clearing price $\widetilde{p}$ should satisfy with probability one

$$\sum_{n=1}^{N} X_n \left(\widetilde{p}, \widetilde{i}_n\right) + Y \left(\widetilde{p}, \widetilde{S}\right) = \widetilde{m}. \tag{1}$$

To emphasize the dependence of the market-clearing price on the strategies of the traders we write

$$p = p(X, Y),\ x_n = x_n(X, Y),\ y = y(X, Y),$$

where X is the vector of strategies of value-informed traders defined by $X = (X_1, ..., X_N)$ and Y is the strategy of the supply-informed trader.

The traders are risk neutral and maximize expected profits. The profits of the value-informed trader n and supply-informed trader are, respectively, given by

$$\widetilde{\pi}_n^{PI} = (\widetilde{v} - \widetilde{p}(X, Y))\, \widetilde{x}_n(X, Y),\quad \widetilde{\pi}^{SI} = (\widetilde{v} - \widetilde{p}(X, Y))\, \widetilde{y}(X, Y).$$

With these notations, following Kyle (1989) we can proceed to define a rational expectations equilibrium in our setup.

Definition 1 *An imperfectly competitive rational expectations equilibrium is defined as a vector (X, Y, p), where X is a vector of strategies of the value-informed agents*

$X = (X_1, ..., X_N)$, Y is a strategy of the supply-informed agent and p is the equilibrium price such that the following conditions hold:

1. For all $n = 1, ..., N$ and for any alternative strategy vector X' differing from X only in the n^{th} component X_n, the strategy X yields a higher profit than X':

$$E_n\left[(\widetilde{v} - \widetilde{p}(X,Y))\widetilde{x}_n(X,Y)|\ \widetilde{p}(X,Y) = p,\ \widetilde{i}_n = i\right] \geq$$
$$E_n\left[(\widetilde{v} - \widetilde{p}(X',Y))\widetilde{x}_n(X',Y)|\ \widetilde{p}(X',Y) = p,\ \widetilde{i}_n = i\right].$$

2. For any alternative strategy Y' the strategy Y yields a higher profit than Y' :

$$E\left[(\widetilde{v} - \widetilde{p}(X,Y))\widetilde{y}(X,Y)|\ \widetilde{p}(X,Y) = p,\ \widetilde{S} = S\right] \geq$$
$$E\left[(\widetilde{v} - \widetilde{p}(X',Y))\widetilde{y}(X,Y')|\ \widetilde{p}(X,Y') = p,\ \widetilde{S} = S\right].$$

3. The price $p = \widetilde{p}(X,Y)$ clears the market (with probability one) i.e.

$$\sum_{n=1}^{N} X_n\left(\widetilde{p}, \widetilde{i}_n\right) + Y\left(\widetilde{p}, \widetilde{S}\right) = \widetilde{m}.$$

This defines a Nash equilibrium in demand functions. Given their private information, traders maximize their conditional expected profits taking into account the effect of their trading on prices and taking as given the strategies other traders use to choose their demand schedules.

We look for a symmetric linear Bayesian Nash Equilibrium as in Kyle (1989), that is, an equilibrium where the strategies X_n and Y are linear functions:

$$X_n\left(\widetilde{p}, \widetilde{i}_n\right) = \alpha^{PI} + \beta^{PI}\widetilde{i}_n - \gamma^{PI}\widetilde{p}, \text{ for any } n = 1, ..., N \text{ and}$$
$$Y\left(\widetilde{p}, \widetilde{S}\right) = \alpha^{SI} + \beta^{SI}\widetilde{S} - \gamma^{SI}\widetilde{p}, \tag{2}$$

where $\alpha^{PI}, \beta^{PI}, \gamma^{PI}, \alpha^{SI}, \beta^{SI}, \gamma^{SI} \in \mathbb{R}$.

With this assumption we can infer from the market clearing condition that the equilibrium price is given by

$$p = \left(N\gamma^{PI} + \gamma^{SI}\right)^{-1}\left(N\alpha^{PI} + \alpha^{SI} + \beta^{PI}\sum_{n=1}\widetilde{i}_n + \left(\beta^{SI} - 1\right)\widetilde{S} - \widetilde{m}\right). \tag{3}$$

Characterization of the Equilibrium

In the following proposition we describe the equations that characterize the symmetric Bayesian-Nash equilibrium. This equilibrium has linear trading and pricing rules and is shown to be unique among all linear, symmetric Bayesian-Nash equilibria. As in most Kyle type models, the linearities are not ex-ante imposed in the agents strategy sets: as long as the informed traders use linear trading strategies, the pricing rule will be linear and vice-versa.

Proposition 1 *If $N(N-2) \geq \frac{\sigma_e^2}{\sigma_v^2}$ there exists a unique linear symmetric equilibrium defined as:*

$$\begin{aligned} X_n\left(\widetilde{p},\widetilde{i}_n\right) &= \alpha^{PI} + \beta^{PI}\widetilde{i}_n - \gamma^{PI}\widetilde{p}, \text{ for any } n = 1,...,N \text{ and} \\ Y\left(\widetilde{p},\widetilde{S}\right) &= \alpha^{SI} + \beta^{SI}\widetilde{S} - \gamma^{SI}\widetilde{p}, \end{aligned}$$

with $\alpha^{PI}, \beta^{PI}, \gamma^{PI}$, $\alpha^{SI}, \beta^{SI}, \gamma^{SI}$ given by

$$\begin{aligned} \alpha^{PI} &= \frac{\sigma_e^2\left(N\left(3N-2\right)\sigma_v^2 + \left(2N-1\right)\sigma_e^2\right)\delta^{1/2}}{2N^2\sigma_v^2\left(N^2\sigma_v^2+\sigma_e^2\right)\left(N\sigma_v^2+\sigma_e^2\right)}\overline{v} + \frac{N\left(N-2\right)\sigma_v^2 - \sigma_e^2}{N\left(N^2\sigma_v^2+\sigma_e^2\right)}\overline{m} \\ \beta^{PI} &= \frac{\delta^{1/2}}{2N\left(N\sigma_v^2+\sigma_e^2\right)} \\ \gamma^{PI} &= \frac{\left(N^2\sigma_v + \left(2N-1\right)\sigma_e^2\right)\delta^{1/2}}{2N^2\sigma_v^2\left(N^2\sigma_v^2+\sigma_e^2\right)} \\ \alpha^{SI} &= -\frac{\left(N-1\right)\left(N^2\sigma_v^2 + \left(2N-1\right)\sigma_e^2\right)\sigma_e^2\delta^{1/2}}{2N^2\sigma_v^2\left(N\sigma_v^2+\sigma_e^2\right)\left(N^2\sigma_v^2+\sigma_e^2\right)}\overline{v} + \frac{N^2\sigma_v^2 + \left(2N-1\right)\sigma_e^2}{N\left(N^2\sigma_v^2+\sigma_e^2\right)}\overline{m} \\ \beta^{SI} &= \frac{N^2\sigma_v^2 + \left(2N-1\right)\sigma_e^2}{2N\left(N\sigma_v^2+\sigma_e^2\right)} \\ \gamma^{SI} &= -\frac{\left(N-1\right)\sigma_e^2\left(N^2\sigma_v^2 + \left(2N-1\right)\sigma_e^2\right)\delta^{1/2}}{2N^2\sigma_v^2\left(N\sigma_v^2+\sigma_e^2\right)\left(N^2\sigma_v^2+\sigma_e^2\right)}, \end{aligned} \tag{4}$$

where

$$\delta \equiv \frac{\left(N(N-2)\sigma_v^2 - \sigma_e^2\right)\left(N^2\sigma_v^2+\sigma_e^2\right)\sigma_S^2}{\left(N-1\right)\sigma_e^2}.$$

The condition $N(N-2) \geq \frac{\sigma_e^2}{\sigma_v^2}$ is similar to the usual condition $N > 2$ in all Kyle-type models. It tells us that we need competition in order to alleviate the asymmetric information problem. In our model, the asymmetric information problem is even more important than in Kyle (1985, 1989) because we have two different types of information that aggregate in prices. The supply-informed agent acts as an informational monopolist trading on the information about supply and thus, he always extracts some rents. However, he also observes the average of the value-informed agents' signals. Since this average is informationally equivalent to observing the whole vector of private signals, he uses this as a private signal about the liquidation value. However, the quality of this signal depends on the homogeneity of the signals received by value-informed traders. The value-informed traders are asymmetrically informed, so increasing their number will make them compete more aggressively against each other and reveal more information. This increased competition will make the average signal more informative and therefore, the supply-informed agent better informed. Consequently, in the case of heterogeneity of the value-informed traders' signals (σ_e^2/σ_v^2 high), we need more competition in order to refine the final information embedded in prices. As we will explain later, we have a bidirectional flow of information between traders (from the supply-informed trader to the value-informed agents and vice-versa). If the signal on liquidation value inferred by the supply-informed agent from prices is poor, this is reflected in his trading, and it alters the information revealed by him about supply. The value-informed agents try to infer this information from prices and the change might lead to a situation where equilibrium fails to exist.

We would like to understand the effects of different types of information on market liquidity, informativeness of prices, price volatility, and the ability of informed traders to exploit their private information. We are first concerned with market liquidity because it has been recognized as an important determinant of market behavior. There are different measures of market liquidity used in the literature: market depth, bid-ask spread and price movement after trade. We will use as a measure of liquidity market depth (as defined by Kyle (1985)), which represents the trading volume needed to move prices one unit. While solving the above system we obtained

$$\gamma = N\gamma^{PI} + \gamma^{SI} = \frac{\left(N^2\sigma_v^2 + (2N-1)\,\sigma_e^2\right)\delta^{1/2}}{2N^2\sigma_v^2\left(N\sigma_v^2 + \sigma_e^2\right)}.$$

On the other hand, from the price equation (3) we can see that an increase (decrease) in the known component of supply by γ induces the price to fall (rise) by one dollar. We use the same measure as Kyle and consequently, γ is our measure of market liquidity. As can be seen, market depth γ has two components that have opposite effect. The first component $N\gamma^{PI}$ is attributed to the value-informed agents trading. This is the amount by which they contribute to a change in the price when each of them trades an additional unit. The more value-informed agents are in the market, the higher the liquidity. Similarly, γ^{SI} is the change in price due to an additional unit of trading by the supply-informed agent. The two components have opposite signs and we thus have a trade-off: the value-informed agents increase market liquidity while the supply-informed agent reduces it.

The fact that γ^{SI} is negative is a very important result in our model and it is a consequence of the mechanism of information transmission through prices. In general, with asymmetric information, prices play a dual role of information aggregation and market clearing. The role of information aggregation played by prices is even more important in our economy with asymmetric and different information. We have two important channels through which information flows: through one channel we have a flow of information about the liquidation value from the value-informed traders towards the supply-informed trader and through the other one we have a flow of information about supply from the supply-informed trader towards the value-informed traders. The supply-informed agent puts a positive weight on price ($\gamma^{SI} < 0$) because when he sees an increase in price, he associates it with good news about the liquidation value (he knows the supply, so the price increase cannot be due to a decrease in supply). This mechanism of information transmission actually triggers a decrease in market liquidity. For one additional unit demanded by a value-informed agent, the price goes up. The supply-informed agent associates it with good news about the liquidation value and increases his demand leading to a even higher increase in price. Since the same volume increases the price more, we may conclude that we have a decrease in market liquidity.

Next, let us investigate how the market depth varies with the parameters of the model: the variance of the liquidity shock σ_S^2, the variance of signals σ_e^2, and the variance of the liquidation value σ_v^2.

Corollary 2 *(i) Market depth is increasing in the variance of liquidity shock* $\widetilde{S}$, σ_S^2.

(ii) Market depth is decreasing in the variance of the error of the signal received by value-informed agents σ_e^2.

(iii) Market depth viewed as a function of the variance of liquidation value σ_v^2 *has an inverted U-shape.*

As we have seen before, the market depth has two components $\gamma = N\gamma^{PI} + \gamma^{SI}$. The first component is the contribution to the market depth of trades executed by value-informed agents while the second one is the contribution to the market depth of trades executed by the supply-informed agent. The two components have opposite effects and thus, the final result on market depth due to the market-making activity of the agents depends on which of the two components dominates. The first result in the Corollary is similar to the previous ones in the literature (Kyle (1985) and other imperfect competition models). It tells us that the higher the variance of the supply (in the other papers - the variance of the noise trading), the easier it is for value-informed agents to hide and therefore to make use of their informational advantage (the volume needed to move the price is higher, and this helps them to trade better on their information without revealing too much of it). In addition, in our model the same is true for the supply-informed agents. If the variance of the liquidity shock (or signal of the supply-informed agent) σ_S^2 is high the supply-informed agent is better camouflaged and can trade more actively on his private information about supply. The second result claims that if the signals of the value-informed agents are very poor, market depth is low. This is due to the fact that when the variance of errors σ_e^2 is high, the signals received by the value-informed traders are very heterogenous, and we have seen that the heterogeneity brings about lower market liquidity. What actually happens is that when the difference in the information between the value-informed agents is small, they will compete more strongly against the supply-informed agent and less among themselves. Once their information becomes very different, i.e. σ_e^2 increases, they will also start competing

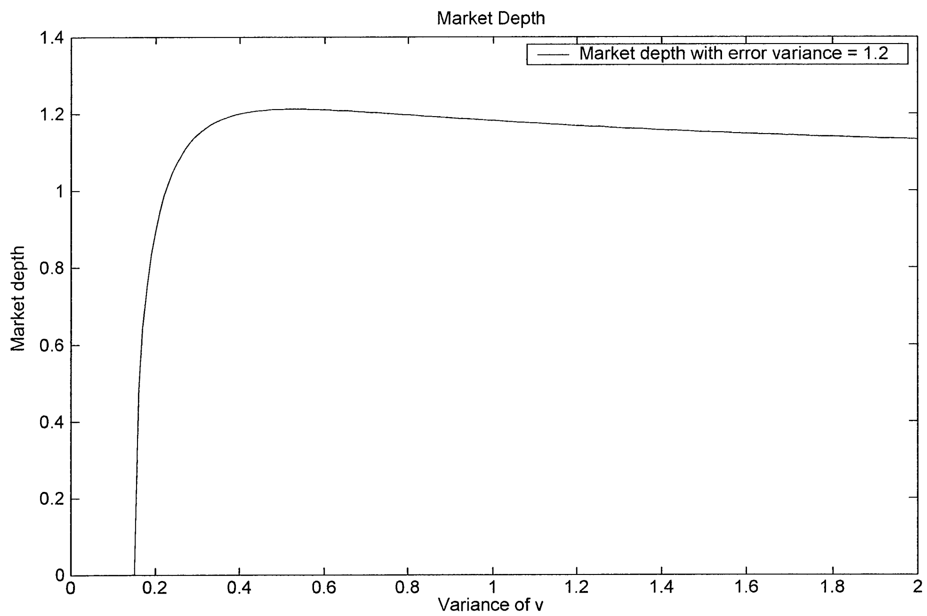

Figure 1: Comparative statics for market depth. Parameters values: $N = 4$, $\sigma_e^2 = 1.2$, $\sigma_S^2 = 2$.

more aggressively against each other. Notice also that these results indicate that the effect on market depth of the trades of value-informed agents dominates the effect of the trades of the supply-informed agent for all values of σ_S^2 or σ_e^2.

As we can also see in Figure 1, market depth has an inverted U-shape. This result differs from the previous results in literature and this difference is triggered precisely by the existence of a supply-informed agent. Where there are only value-informed traders, we have that the higher the variance of the liquidation value, the higher their informational advantage and therefore the lower the market depth. The existence of the supply informed-agent affects the informational advantage of the value-informed agents. If the variance σ_v^2 is small, the average signal about the liquidation value inferred by the supply-informed agent is quite good. So the supply-informed agent can infer the private information of the value-informed agents quite well, thus reducing their informational advantage and inducing an increase of market liquidity. However, as the variance of liquidation value σ_v^2 increases, the informational advantage of the value-informed trader increases substantially offsetting this effect and therefore, market depth decreases.

Once we have determined the equilibrium demand strategies, we can also determine the market clearing price.

Corollary 3 *The equilibrium price is given by*

$$\widetilde{p} = \frac{\sigma_e^2 (2N-1)}{N^2\sigma_v^2 + (2N-1)\sigma_e^2}\overline{v} + \frac{N\sigma_v^2}{N^2\sigma_v^2 + (2N-1)\sigma_e^2}\sum_{n=1}^{N}\widetilde{i}_n - \frac{N\sigma_v^2 (N^2\sigma_v^2 + \sigma_e^2)}{(N^2\sigma_v^2 + (2N-1)\sigma_e^2)\,\delta^{1/2}}\widetilde{S} - \frac{2N\sigma_v^2 (N\sigma_v^2 + \sigma_e^2)}{(N^2\sigma_v^2 + (2N-1)\sigma_e^2)\,\delta^{1/2}}\overline{m} \tag{5}$$

From this corollary we can see that the unconditional expectation of the equilibrium price is

$$E(\widetilde{p}) = \overline{v} - \frac{2N\sigma_v^2 (N\sigma_v^2 + \sigma_e^2)}{(N^2\sigma_v^2 + (2N-1)\sigma_e^2)\,\delta^{1/2}}\overline{m}$$

and it depends on the expected supply $\overline{m}$. If $\overline{m} = 0$, the price is an unbiased estimator of $\overline{v}$, but it is biased if $\overline{m} \neq 0$. We also can see that, as expected, the higher the supply (the expected supply $\overline{m}$, or the realization of the liquidity shock $\widetilde{S}$ observed by the supply-informed agent), the lower the price and the greater the signals received by the value-informed agents, the higher the price.

Also note that a change in the different components of the supply has a different impact on price. A change in the known part of supply $\overline{m}$ is absorbed by the agents through the quantity demanded in a proportion of $\frac{N-1}{N}$ (we have seen while calculating the strategies that $\alpha = N\alpha^{PI} + \alpha^{SI} = g(N, \sigma_v^2, \sigma_e^2) + \frac{(N-1)}{N}\overline{m}$, where $g(N, \sigma_v^2, \sigma_e^2)$ is a function which does not depend on $\overline{m}$) and only $\frac{1}{N}$ is reflected in price. Similarly, half of a shock in the component of supply known to supply-informed agent $\widetilde{S}$ is absorbed by this agent through his demand and is partly reflected in price. As I have already explained, the supply-informed trader has a monopolist position and extracts rents that amount to half of the unknown component of supply.

Market Indicators

In what follows, we study the implications the existence of a supply-informed agent has for the market performance. We compute some market indicators: volatility of prices, informativeness of prices and expected profits of different market participants and characterize them with respect to the variance of the liquidation value of the asset.

Corollary 4 *The price volatility, measured as the variance of price, is*

$$Var(\widetilde{p}) = \frac{N^3(N-2)\left(\sigma_v^2\right)^2 + 2N^2(N-2)\,\sigma_v^2\sigma_e^2 - \left(\sigma_e^2\right)^2}{\left(N(N-2)\sigma_v^2 - \sigma_e^2\right)}\left(\frac{N\sigma_v^2}{N^2\sigma_v^2 + (2N-1)\,\sigma_e^2}\right)^2.$$

As in the case where there is no supply-informed agent, we find that the volatility of prices does not depend on the noise in supply. If the noise in supply increases all the agents - both the value-informed and the supply-informed - trade more aggressively making better use of their particular informational advantage. We also find that price volatility has a U shape with respect to the variance of the liquidation value of the asset, σ_v^2. Looking at the way the information is incorporated in prices (see Equation 5) we observe that the weight associated with the information of the value-informed agents increases with σ_v^2, while the weight associated with the information of the supply-informed agent decreases.[2] For small values of σ_v^2, the effect this has on the information about supply revealed in prices dominates the one on information about the liquidation value of the asset. However, when σ_v^2 increases, the value-informed traders will trade more aggressively, revealing more information and thus increasing the volatility of prices. It is interesting to note that if competition increases, the range in which the volatility of prices is decreasing in σ_v^2 shrinks and we recover the result from the case without a supply-informed trader i.e. the higher the variance of the liquidation value of the asset, the higher the volatility of prices. As a result, in a market where there are enough value-informed agents, the volatility of prices increases.

[2]This weight is actually the intensity of trading on information divided by the market depth.

Next, we would like to find out the amount of private information - both about the liquidation value and supply - that is revealed through prices. We thus define the information content of prices as the difference between the prior variance of the payoff and the variance conditional on prices. Using the normality assumptions we obtain the expression presented in the following Corollary:

Corollary 5 *The information content of prices is*

$$Var(\widetilde{v}) - Var(\widetilde{v}|\widetilde{p}) = \frac{N^2(\sigma_v^2)^2 (N(N-2)\sigma_v^2 - \sigma_e^2)}{N^3(N-2)(\sigma_v^2)^2 + 2N^2(N-2)\sigma_v^2\sigma_e^2 - (\sigma_e^2)^2}.$$

As with the previous Corollary, we also obtain here that price efficiency or the information content of prices does not depend on the variance of supply shock $\widetilde{S}$. Moreover, we obtain that informativeness of prices increases with respect to the variance of the liquidation value σ_v^2 and decreases with respect to the variance σ_e^2. These results tells us that when it is difficult to predict the liquidation value or when the signals of value-informed agents are poor, prices play a very important role in information aggregation. While these results, are qualitatively similar to the case without supply-informed agent, as we will see later, they are quantitatively different.

Let us turn to the expected volume traded by the value-informed agent and supply-informed agent, respectively.

Corollary 6 *The expected volume traded by a value-informed agent is*

$$E(|x_n|) = \frac{2(N-1)\sigma_v^2\overline{m}}{N^2\sigma_v^2 + \sigma_e^2} + \frac{\left(\frac{2}{\pi}\right)^{1/2}}{4N^2}\left(\frac{\left(N^2\sigma_v^2+\sigma_e^2\right)^2 + N\left(N\sigma_v^2+\sigma_e^2\right)^2}{\left(N\sigma_v^2+\sigma_e^2\right)^2\left(N^2\sigma_v^2+\sigma_e^2\right)^2}\left(\sigma_v^2+\sigma_e^2\right)\delta + \sigma_S^2\right).$$

The expected volume traded by the supply-informed trader is

$$E(|y|) = \frac{2\left(N\sigma_v^2+\sigma_e^2\right)\overline{m}}{\left(N^2\sigma_v^2+\sigma_e^2\right)} + \left(\frac{1}{8\pi}\right)^{1/2}\sigma_S^2\left(1 + \frac{(N-1)\sigma_e^2\left(N(N-2)\sigma_v^2-\sigma_e^2\right)\left(\sigma_v^2+\sigma_e^2\right)}{N\left(N^2\sigma_v^2+\sigma_e^2\right)\left(N\sigma_v^2+\sigma_e^2\right)^2}\right).$$

The expected volumes traded by the value-informed agents and the supply-informed agent depend positively on the expected supply $\overline{m}$ and the variance of the supply shock σ_S^2. However, both the effects of an increase in σ_S^2 and in $\overline{m}$ are stronger in the case of supply-informed trader. This is the role we actually wanted the supply-informed agent to play. Since he has information about supply he captures a big part of the shocks. In the previous literature, where agents only had information about the liquidation value, the trading volume neither depended on the variance of the liquidation value nor on the variance of the errors. In our case, they do depend and moreover, when the known component in supply $\overline{m}$ is different from 0, the comparative statics with respect to the variance of the liquidation value σ_v^2 and the one of the error σ_e^2 are ambiguous. Where the known component in supply $\overline{m}$ is equal to zero, we find that the expected volume traded by the informed agents increases with respect to the variance of liquidation value σ_v^2 and decreases with the variance of the errors σ_e^2. The reasons are the same as before: the higher the variance of liquidation value, the better the informational advantage of the value-informed traders, so the higher the expected volume. Also, the higher the variance of errors, the more heterogeneous are the signals received by the value-informed traders. This implies lower quality of price as a signal about the supply, and therefore lower volume of trading by value-informed traders. On the other hand, from the point of view of the supply-informed agent both high variance of liquidation value σ_v^2 and high variance of the errors σ_e^2 imply high heterogeneity of the signals of value-informed traders and this implies bad quality of price as a signal about the liquidation value. However, heterogeneity makes the value-informed traders trade more aggressively against one another. As a result, the expected volume traded by the supply-informed trader is inverted U-shaped, the shape being detemined by which of the above mentioned effects dominates.

We next compute the unconditional profits for all agents.

Corollary 7 *The unconditional expected profit of the n^{th} value-informed agent is*

$$\begin{aligned}\Pi_n^{PI} &= E\left(\widetilde{\pi}_n^{PI}\right) = \frac{\sigma_v^2 \delta^{1/2} (N-1)\sigma_e^2}{2N\left(N^2\sigma_v^2 + (2N-1)\sigma_e^2\right)\left(N\sigma_v^2+\sigma_e^2\right)}\left(\frac{N\left(N\sigma_v^2+\sigma_e^2\right)}{(N(N-2)\sigma_v^2-\sigma_e^2)} - \right.\\ &\left.\frac{(N-1)\sigma_e^2}{\left(N^2\sigma_v^2+\sigma_e^2\right)}\right) + \frac{(N-1)}{\left(N^2\sigma_v^2+\sigma_e^2\right)}\frac{2N\sigma_v^2\left(N\sigma_v^2+\sigma_e^2\right)}{\left(N^2\sigma_v^2+(2N-1)\sigma_e^2\right)\delta^{1/2}}\overline{m}^2.\end{aligned}$$

The unconditional profit of the supply-informed agent is

$$\Pi^{SI} = E\left(\widetilde{\pi}^{SI}\right) = \frac{\delta^{1/2}\left(N-1\right)\sigma_e^2\sigma_v^2}{2\left(N^2\sigma_v^2+\left(2N-1\right)\sigma_e^2\right)}\left(\frac{\left(N-1\right)\sigma_e^2}{\left(N^2\sigma_v^2+\sigma_e^2\right)\left(N\sigma_v^2+\sigma_e^2\right)}+\frac{N}{\left(N(N-2)\sigma_v^2-\sigma_e^2\right)}\right)+\frac{2N\sigma_v^2\left(N\sigma_v^2+\sigma_e^2\right)}{\left(N^2\sigma_v^2+\left(2N-1\right)\sigma_e^2\right)\delta^{1/2}}\frac{\left(N\sigma_v^2+\sigma_e^2\right)}{\left(N^2\sigma_v^2+\sigma_e^2\right)}\overline{m}^2.$$

As we expected, allowing the supply-informed agent to behave strategically allows him to make positive profits by comparison with the case of perfect competition where he makes zero profits. Notice also that since the value-informed traders always absorb $\frac{1}{2N}$ of the shock S, it is actually the different information that they receive that makes them have different profits. Notice that the non-monotonicity with respect to the variance of liquidation value is transmitted here also, both expected profits having a U shape.

We also want to see what the impact of changes in supply is on the equilibrium price and the quantity demanded by the different agents. Similar to Gennotte and Leland (1990), we study the two following cases: a supply increase known to all agents $\overline{m}$, and a supply increase known only to supply-informed agent $\widetilde{S}$.

Corollary 8 *A positive shock in supply known to all the agents $\overline{m}$ leads to an increase in the demand of both type of agents, a decrease in the equilibrium price and therefore, to an increase in the expected profits of both type of agents.*

As expected, an increase in the supply known to all agents makes them adjust their demands in accordance with the existing supply, and it also leads to a decrease of the equilibrium price. Here, we find that the value-informed agents always absorb a greater proportion of the shock in supply $\overline{m}$.

Corollary 9 *A positive shock in the component of supply $\widetilde{S}$, known to the supply-informed agent, decreases the equilibrium price and increases the quantities demanded both by the value-informed and supply-informed agents.*

As expected, in the case of a positive shock in the supply $\widetilde{S}$, the supply-informed agent increases his demand, making use of the private information he has. Moreover, the increase in supply (due to a positive shock in $\widetilde{S}$) absorbed by the supply-informed agent is N times higher than the increase of supply absorbed by any value-informed agents. An interesting result is that the supply-informed agent always absorbs half of the unobservable shock in supply, the other half being absorbed by value-informed agents. This result resembles somewhat the one obtained by Röell (1990), and is explained by the fact that the supply-informed trader acts as a monopolist, extracting half of the rents.[3] Notice that in our model, the supply-informed trader always extracts half of the rents despite the fact that they submit limit orders, while in Röell (1990) this was only possible if either the number of brokers-dealers increased significantly, or the brokers-dealers submitted market orders.

Equilibrium without a Supply-Informed Agent

In order to see the effects of different types of information on market liquidity, informativeness of prices, price volatility, and the ability of informed traders to exploit their private information, we need to provide a benchmark for making a comparison with the equilibrium characterized in the previous section. A first step will be to see how the presence in the market of a supply-informed agent affects all these market structure indicators. For this purpose we first characterize, in a similar manner, the equilibrium without a supply-informed agent. Notice that this model is a version of Kyle's (1989) model with the difference that we do not have uninformed agents and we replace the noise agents by a random supply.

Proposition 10 *There is a unique linear symmetric equilibrium defined as:*

$$X_{I,n}\left(\widetilde{p},\widetilde{i}_n\right) = \alpha_I + \beta_I \widetilde{i}_n - \gamma_I \widetilde{p}, \text{ for any } n = 1, ..., N$$

[3] In a model that examines the effects of dual trading, Röell (1990) considers several broker-dealers who have better information about uninformed traders than the market maker.

where $\alpha_I, \beta_I, \gamma_I$ are given by

$$\begin{aligned}
\alpha_I &= \frac{2\sigma_e^2}{N\sigma_v^2}\left(\frac{(N-2)\,\sigma_S^2}{N(N-1)\,\sigma_e^2}\right)^{1/2}\overline{v}+\frac{(N-2)}{N(N-1)}\overline{m}\\
\beta_I &= \left(\frac{(N-2)\,\sigma_S^2}{N(N-1)\,\sigma_e^2}\right)^{1/2}\\
\gamma_I &= \frac{N\sigma_v^2+2\sigma_e^2}{N\sigma_v^2}\left(\frac{(N-2)\,\sigma_S^2}{N(N-1)\,\sigma_e^2}\right)^{1/2}.
\end{aligned}$$

Similarly to the case with a supply-informed agent, we proceed with the calculations of the equilibrium price and equilibrium quantities traded by the value-informed agent.

Corollary 11 *The equilibrium price when there is no supply-informed agent is*

$$\begin{aligned}
\widetilde{p}_I &= \frac{2\sigma_e^2}{N\sigma_v^2+2\sigma_e^2}\overline{v}+\frac{\sigma_v^2}{N\sigma_v^2+2\sigma_e^2}\sum_{n=1}^{N}\widetilde{i}_n-\frac{\sigma_v^2}{N\sigma_v^2+2\sigma_e^2}\left(\frac{N(N-1)\,\sigma_e^2}{(N-2)\,\sigma_S^2}\right)^{1/2}\widetilde{S}\\
&\quad -\frac{\sigma_v^2}{(N\sigma_v^2+2\sigma_e^2)(N-1)}\left(\frac{N(N-1)\,\sigma_e^2}{(N-2)\,\sigma_S^2}\right)^{1/2}\overline{m}.
\end{aligned}$$

Notice that the price is here also an unbiased estimator of $\widetilde{v}$ if and only if $\overline{m}=0$. Next we compute the same market indicators we computed for the economy with a supply-informed agent.

Corollary 12 *The market indicators for an economy without a supply-informed agent are the following:*

1) The price volatility, measured as the variance of price, is

$$Var(\widetilde{p}_I)=N\left(\frac{\sigma_v^2}{N\sigma_v^2+2\sigma_e^2}\right)^2\left(N\sigma_v^2+\frac{(2N-3)}{(N-2)}\sigma_e^2\right).$$

2) The information content of prices is

$$Var(\widetilde{v})-Var(\widetilde{v}|\,\widetilde{p}_I)=N(\sigma_v^2)^2(N-2)\left(N(N-2)\,\sigma_v^2+(2N-3)\,\sigma_e^2\right)^{-1}.$$

3) The expected volume traded by a value-informed agent is

$$E\left(|x_{I,n}|\right)=\frac{1}{N}\overline{m}+\left(\frac{2}{\pi}\right)^{1/2}\frac{(N-1)}{N^2}\sigma_S^2.$$

4) The expected profit of a value-informed agent is

$$\Pi_{I,n}^{PI} = E\left(\widetilde{\pi}_{I,n}^{PI}\right) = E\left(\left(\widetilde{v} - \widetilde{p}_I\right)\widetilde{x}_n\right) = \frac{\sigma_v^2}{N\left(N\sigma_v^2 + 2\sigma_e^2\right)}\left(\frac{N\left(N-1\right)\sigma_e^2}{\left(N-2\right)\sigma_S^2}\right)^{1/2}\left(\overline{m}^2 + \sigma_S^2\right).$$

Comparison of Market Indicators

We now compare the market indicators in two cases: one in which there is a supply-informed agent, and one where there is none. Let us first study the effect the presence of the supply-informed agent has on market depth. We have that

$$\gamma \equiv N\gamma^{PI} + \gamma^{SI} = \frac{\left(N^2\sigma_v^2 + \left(2N-1\right)\sigma_e^2\right)\sigma_S}{2N^2\sigma_v^2\left(N\sigma_v^2 + \sigma_e^2\right)\sigma_e}\left(\frac{\left(N(N-2)\sigma_v^2 - \sigma_e^2\right)\left(N^2\sigma_v^2 + \sigma_e^2\right)}{\left(N-1\right)}\right)^{1/2}$$

the market depth where there is a supply-informed agent and

$$\gamma^I \equiv N\gamma_I = \frac{\left(N\sigma_v^2 + 2\sigma_e^2\right)\sigma_S}{\sigma_v^2\sigma_e}\left(\frac{\left(N-2\right)}{N\left(N-1\right)}\right)^{1/2}$$

the market depth where there is none. As we can see in Figure 2, market depth is less where we have a supply-informed agent in the market $\gamma < \gamma^I$. This result is quite intuitive if one considers that the supply-informed agent plays a dual role in the market. First, he reveals himself a part of his information in the process of trading. Second, by having the information about supply, he makes the value-informed agents reveal more of their information. Notice that our agents observe only one type of information, but they place limit orders and therefore, through the price, they also trade on the information of the other market participants. This is similar to the literature on dual trading Röell (1990), Fishman and Longstaff (1992), and Sarkar (1995) where dealer-brokers together with information about the liquidation value are able to observe a component of the order flow. However, in our model, with imperfect competition and limit orders, the presence of the supply-informed trader plays a more complex role as we can see by studying the other market indicators.

Subrahmanyam (1991) also finds that market liquidity decreases when the amount of information in the market increase (when the number of informed traders increases) and the market maker is risk averse. Although the decrease in market

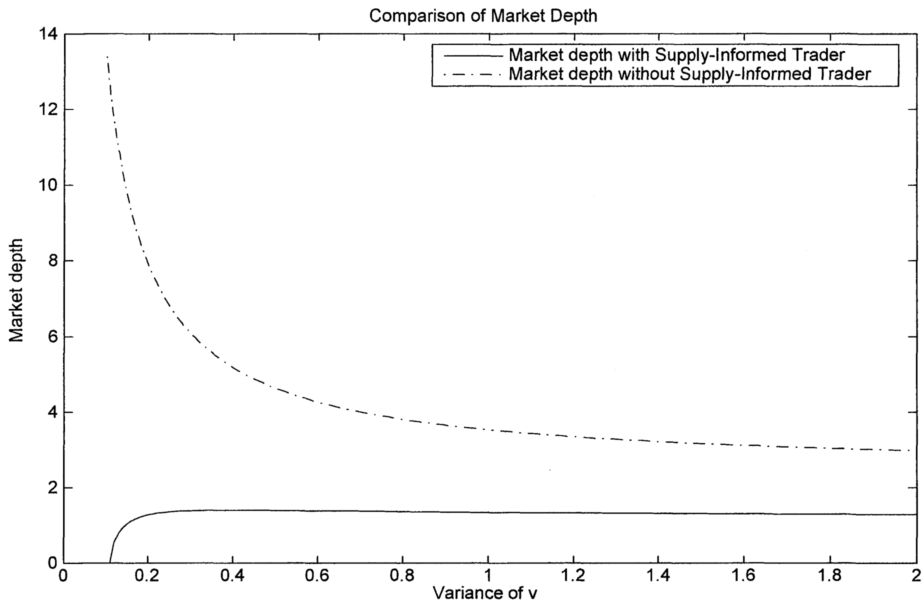

Figure 2: Comparison of market depth with and without supply-informed trader. Parameters values: $N = 4$, $\sigma_e^2 = 0.7$, $\sigma_S^2 = 2$.

liquidity is due to the different types of information, our result is similar to Subrahmanyam's (1991) findings. The similitarity is caused by the fact that the supply-informed agent is risk neutral, but he behaves strategically. Thus the role played by him in the economy is similar to the one played by the risk-averse market maker in Subrahmanyam's (1991) model. Still, if we increase the number of value-informed traders, we once again find the increase in market liquidity obtained by Kyle (1985, 1989) and the subsequent literature.

Finally, this decrease in the market liquidity in the presence of the supply-informed agent captures the intuition of Glosten and Milgrom (1985), that more information in the market decreases market liquidity. In their model, they use the bid-ask spread as a measure of liquidity (low liquidity being equivalent to high bid-ask spread), and an increase in the number of informed agents increases the bid-ask spread.

We also find that when there is a supply-informed trader in the market, value-informed traders trade more aggressively on their private information $\left(\beta^{PI} > \beta_I\right)$

and they devote less to market-making activity[4]

$$\omega^{PI}=\frac{\sigma_e^2\left(N\left(3N-2\right)\sigma_v^2+\left(2N-1\right)\sigma_e^2\right)\delta^{1/2}}{2N^2\sigma_v^2\left(N^2\sigma_v^2+\sigma_e^2\right)\left(N\sigma_v^2+\sigma_e^2\right)}<\omega^I=\frac{2\sigma_e^2}{N\sigma_v^2}\left(\frac{\left(N-2\right)\sigma_S^2}{N\left(N-1\right)\sigma_e^2}\right)^{1/2}.$$

The inside information allows value-informed agents to make gains at the expense of market makers. However, when there is a supply-informed agent who has the ability to disentangle the order flow originated by value-informed agents from a shock in supply, the advantage of the value-informed agent diminishes and therefore, his market-making gains. A part of the gains that the value-informed agents made from market-making activity is now made by the supply-informed agent. As we have already seen, value-informed agents put a greater weight on market-making activity than the supply-informed agent does. This tells us that a dealer, even though he may have information about supply, faces strong competition in market-making from the other value-informed traders. This induces also another interesting result that concerns the volume of trading. We have seen that the volume of trading of value-informed traders where there is no supply-informed agent in the market depends only on the number of informed agents and the variance of the shock in supply. However, where there is a supply-informed agent in the market, the volume of trading depends positively on the variance of the liquidation value. As we can see in Figure 3, when the variance of liquidation value is small, the volume of trading by value-informed traders is smaller in the case when there is a supply-informed trader. As the variance of the liquidation value increases, the volume of trading by value-informed traders increases when there is a supply-informed trader in the market. So our model explains one of the stylized facts about volume: the higher the asymmetry between's trader information, the greater the volume of trading.

Proposition 13 *The presence of the supply-informed agent in the market leads to higher volatility of prices, lower informativeness of prices and lower expected profits by the value-informed agents (when $\overline{m}=0$) .*

[4]The intensity of trading and the intensity of the market making activity are defined in a similar way to the literature as the coefficients of the signals (private signal and price) minus the average signals.

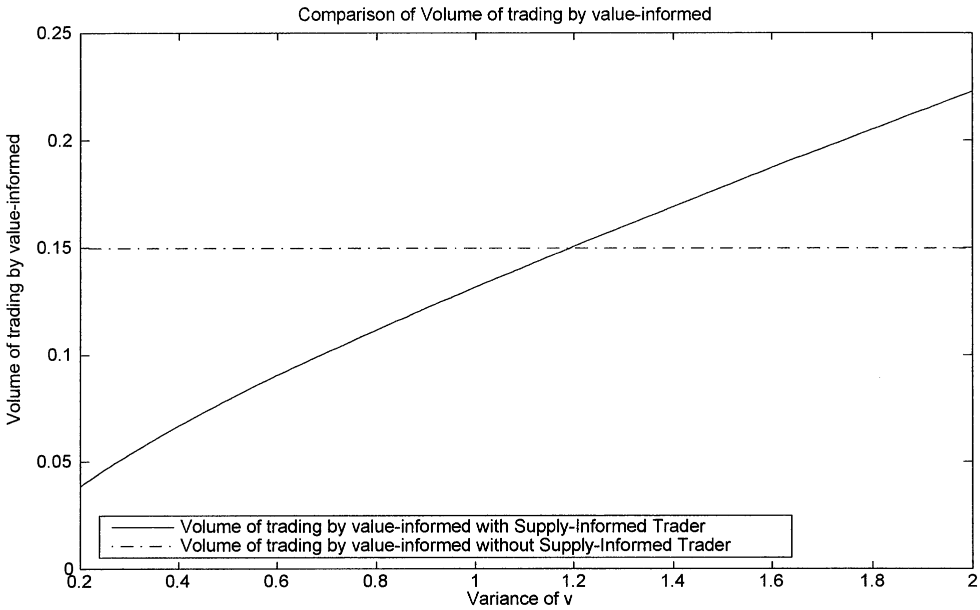

Figure 3: Comparison of volume of trading by the value-informed traders when there is one or no supply informed trader. Parameters values: $N = 4$, $\sigma_e^2 = 1$, $\sigma_S^2 = 2$.

As pointed out above, one should note that the results concerning the trading volume, volatility of prices, informativeness of prices and expected profits by the value-informed agents are very different from the ones in the dual trading literature (Röell (1990), and Sarkar (1995)). As a result of the fact that both type of traders infer the others' information, the reduction in the market depth no longer offsets the impact on order flow. A first implication is that, unlike the other papers, the price informativeness and the volatility of prices are affected by the presence in the market of the supply-informed trader.

The fact that the volatility of prices increases where is a supply-informed agent (see Figure 4), is due to two factors. First, the existence in the market of the information about supply forces value-informed agents to reveal more of their information. Second, the supply-informed trader is also revealing information about the supply and the more information is reavealed in prices, the more volatile the prices are. This can also be seen analyzing the price equations in both cases (the shock in supply affects the price more where there is a supply-informed agent).

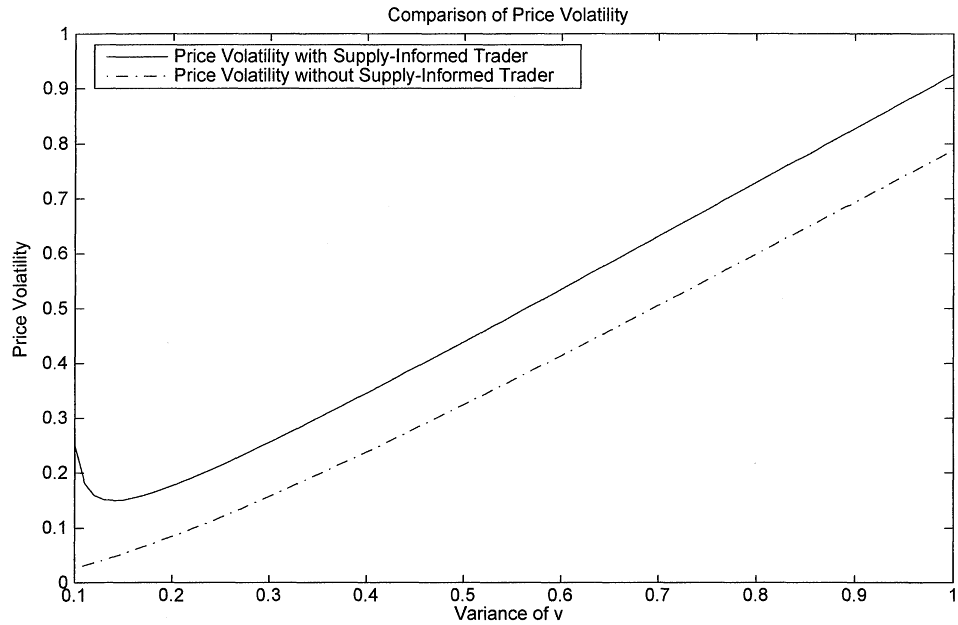

Figure 4: Comparison of price volatility when there is one or no supply informed trader. Parameters values: $N = 4$, $\sigma_e^2 = 0.7$, $\sigma_S^2 = 2$.

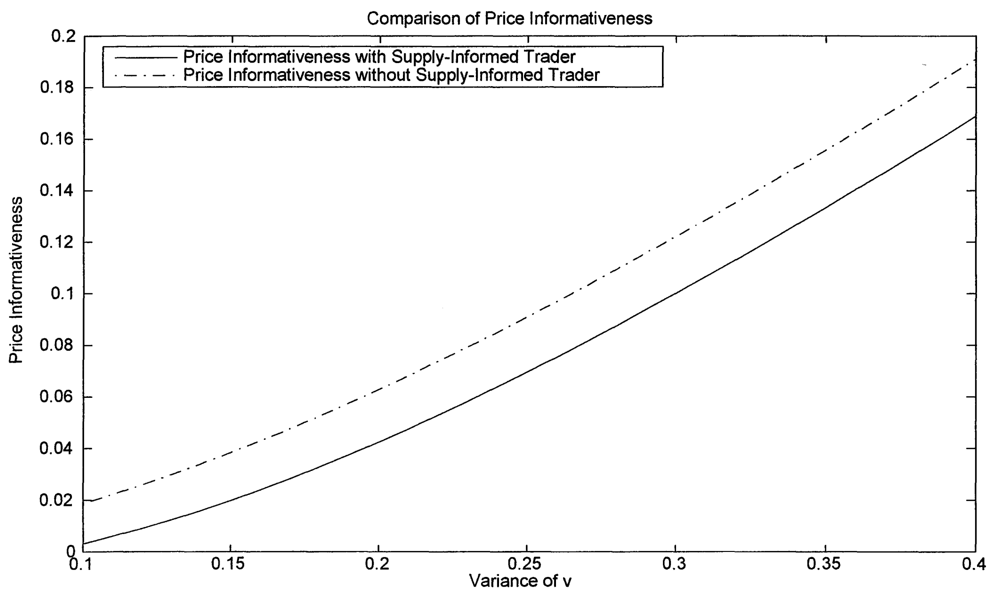

Figure 5: Comparison of price informativeness when there is one or none supply informed trader. Parameters values: $N = 4$, $\sigma_e^2 = 0.7$, $\sigma_S^2 = 2$.

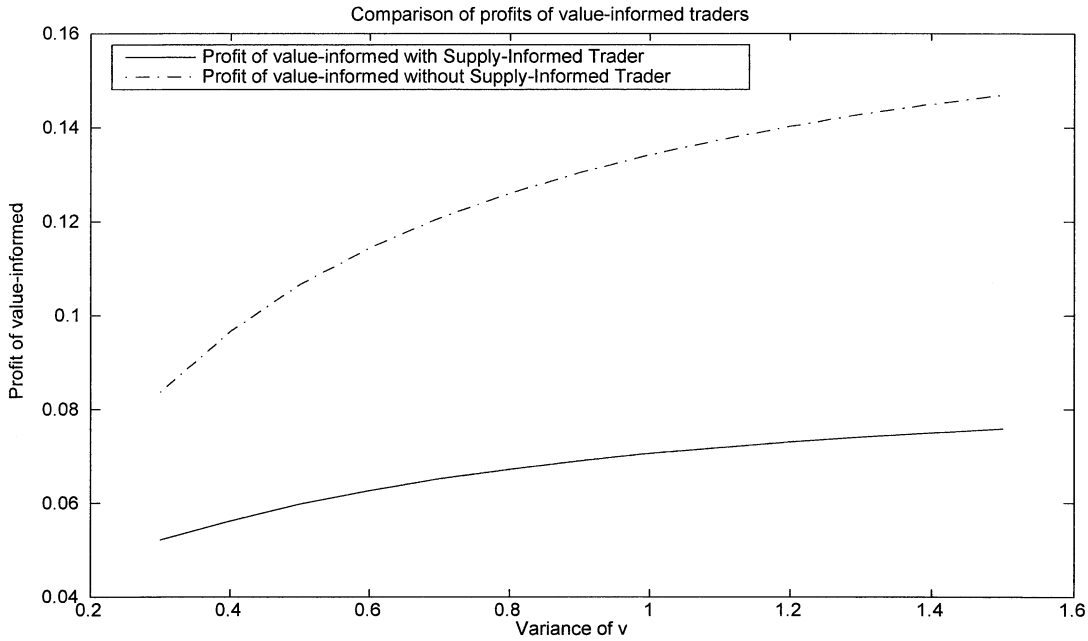

Figure 6: Comparison of expected profits of value-informed when there is one or no supply informed trader. Parameters values: $N = 4$, $\sigma_e^2 = 0.7$, $\sigma_S^2 = 2, \overline{m} = 0$.

As it can be seen in Figure 5, the price informativeness decreases when there is a supply-informed agent in the market. On the one hand, we found that the volatility of prices increases. On the other hand, the intensity of trading on private information also decreases, thus affecting the covariance between the price and liquidation value. However, this effect is not strong enough to offset for the big increase in volatility, and therefore the price informativeness is reduced. Notice that despite more information being aggregated in price, this does not imply that the prices are more informative about the liquidation value of the asset. This is so because the price aggregates two types of information: about liquidation value of the asset and about supply. Of course, when a trader uses the price as a signal together with another signal (either about liquidation value or about supply) the price reveals more information to him.

Finally, we study the expected profits of the value-informed traders. Where the known component of supply $\overline{m}$ is 0, the expected profits of the value-informed traders decrease when there is a supply-informed trader (Figure 6). Despite of the fact that total expected profits increases when there is a supply-informed trader, the biggest

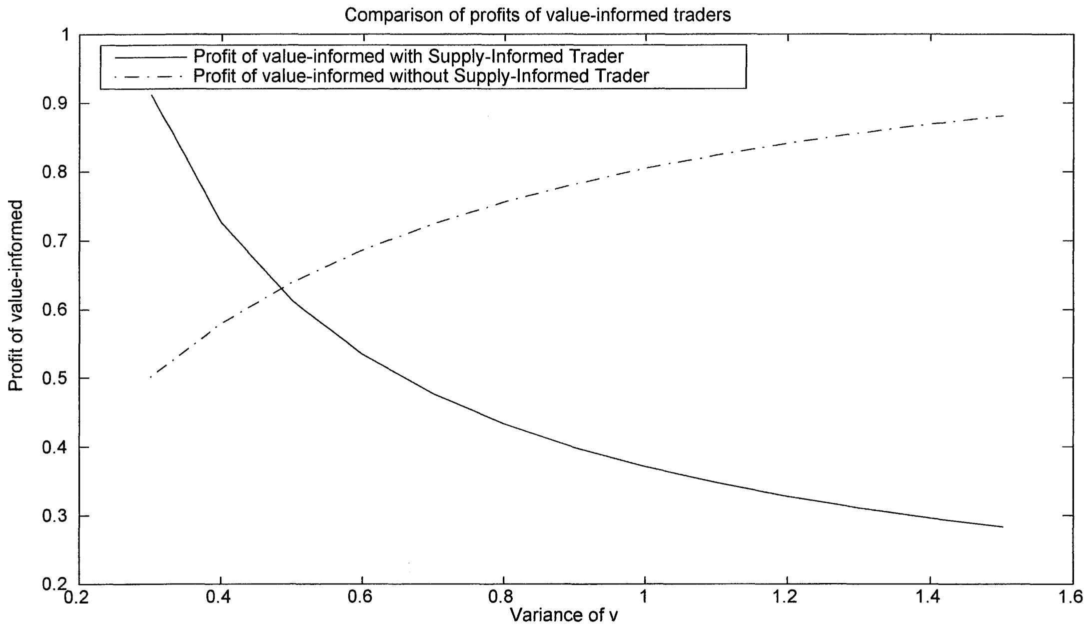

Figure 7: Comparison of expected profits of value-informed when there is one or no supply informed trader. Parameters values: $N = 4$, $\sigma_e^2 = 0.7$, $\sigma_S^2 = 2$.

part of this profits is made now by the supply informed trader.[5] When the known component of supply $\overline{m}$ is different from 0, the profit of the value-informed traders when there is a supply-informed trader is higher for small values of the variance of the liquidation values (Figure 7).

Conclusions

In this paper we have presented a model of insider trading where the agents might have information either about prices or about supply. This information is aggregated and partially revealed through the equilibrium price, so the agents will end up with more information than they initially possess. Our goal is twofold. First we try

[5]We have not modelled the noise traders in the model, but if we had done so this increase in total profits will occur at the expense of the noise traders. Therefore, unlike in Röell (1990) and Sarkar (1995), the transaction costs of the noise traders when there is a supply-informed trader would increase.

to understand how the presence in the market of a supply-informed agent and the interaction with value-informed agents can change the behavior of value-informed agents and the structure of the market. Then, we see how the shocks in different components of supply can alter the market structure, price formation and the behavior of the agents, and therefore the impact of these shocks on the equilibrium outcome.

We consider an imperfectly competitive rational expectations setup and characterize the Bayesian Nash equilibrium in demand schedules. Allowing the supply-informed agent to behave strategically, he makes positive profits (unlike in the perfect competitive case) and increases the amount of information revealed in prices. We see that he has a dual role in inducing information transmission in the market: first because he has superior information (which he reveals in the trading process) and second, because he urges value-informed agent to reveal more of their information. However, the most important consequence of his presence in the market is that he decreases market liquidity (this outcome being brought about by the strategic behavior and the mechanism of information transmission through prices). Using a Kyle-type model we find a similar result to Glosten and Milgrom (1985) (i.e. that more information in the market decreases market liquidity). Modelling the dealer as a supply-informed trader who behaves strategically helps us to link the two strands of the literature.

We also study how market performance is affected in our model by the quality of information received by the agents. The comparative statics results regarding market liquidity measured as market depth tell us that it decreases in the variance of the error of the signal received by value-informed agents, increases in the variance of the supply shock known only by the supply-informed agent and has an inverted U shape with respect to the variance of the liquidation value. Thus, the presence of a supply-informed agent induces non-monotonicity of the market liquidity and other market indicators with respect to the variance of the liquidation value. Comparing the market indicators in our model with the ones in the benchmark case (where there is no supply-informed agent), we conclude that the supply-informed agent does indeed have an important effect. We find that the market depth, informativeness of prices and trading intensity of value-informed agents decrease while volatility of prices increase.

To complete the analysis, we also consider the case when the supply-informed agent has only information about a component of supply. This setup is similar to the one in Gennotte and Leland (1990), where the supply has three components: a component known by everyone, a component known by the supply-informed agent and another one known by nobody. The numerical analysis we have performed for this case suggests a similar pattern. However, in this case the supply-informed agent will not always put a positive weight on price. Since he can no longer perfectly disentangle the two factors that might affect prices (the news about the liquidation value of the asset revealed by the value-informed agents or a shock in the unknown component of supply), he will no longer have the same effect on market liquidity. However, for relative high variance of the known component in supply relative to the unknown component L, $\frac{\sigma_S^2}{\sigma_L^2}$ the result we have obtained here will still hold.

Appendix

Lemma A.1 *In a symmetric linear equilibrium* $N\gamma^{PI} + \gamma^{SI} \neq 0$.

Proof. We look for a symmetric linear equilibrium. Therefore, we use the linear strategies defined in (2) and we can write the market clearing condition (1) as it follows:

$$N\alpha^{PI} + \beta^{PI} \sum_{n=1}^{N} \widetilde{i}_n - N\gamma^{PI}\widetilde{p} + \alpha^{SI} + \beta^{SI}\widetilde{S} - \gamma^{SI}\widetilde{p} = \overline{m} + \widetilde{S}. \tag{6}$$

We define $\gamma \equiv N\gamma^{PI} + \gamma^{SI}$ and $\alpha \equiv N\alpha^{PI} + \alpha^{SI}$. Using these definitions, the market clearing condition can be written as

$$\alpha + \beta^{PI} \sum_{n=1}^{N} \widetilde{i}_n - \gamma\widetilde{p} - (1 - \beta^{SI})\widetilde{S} = \overline{m}.$$

We want to prove that $\gamma \neq 0$. Let us suppose that $\gamma = 0$. Then, the above condition becomes

$$\alpha + \beta^{PI} \sum_{n=1}^{N} \widetilde{i}_n - (1 - \beta^{SI})\widetilde{S} = \overline{m}.$$

Since $\widetilde{i}_n$, $n = 1, ..., N$ are independent of $\widetilde{S}$, it results that $\beta^{PI} = 0$. Plugging it in the above equation we obtain that

$$\alpha - (1 - \beta^{SI})\widetilde{S} = \overline{m},$$

which cannot be satisfied because α and $\overline{m}$ are real numbers and $\widetilde{S}$ is a random variable. We obtained therefore, a contradiction. ■

Lemma A.2 *In a symmetric linear equilibrium the optimal demand for the value-informed trader n and for the supply-informed trader are, respectively,*

$$x_n\left(\widetilde{p}, \widetilde{i}_n\right) = ((N-1)\gamma^{PI} + \gamma^{SI})\left[E\left(\widetilde{v}\left|\widetilde{p}, \widetilde{i}_n\right.\right) - \widetilde{p}\right] \tag{7}$$

$$y(\widetilde{p}, \widetilde{S}) = N\gamma^{PI}\left[E\left(\widetilde{v}\left|\widetilde{p}, \widetilde{S}\right.\right) - \widetilde{p}\right] \tag{8}$$

with $\gamma^{PI} > 0$, *and* $(N-1)\gamma^{PI} + \gamma^{SI} > 0$.

Proof. Let us first determine the optimal demand for the value-informed traders. The value-informed trader n considers the other players' strategies as given by (2). As a result, he is facing the following residual demand:

$$p = \frac{\alpha - \alpha^{PI} + \beta^{PI} \sum_{j \neq n} \widetilde{i}_j - (1 - \beta^{SI})\widetilde{S} - \overline{m}}{(N-1)\gamma^{PI} + \gamma^{SI}} + \frac{x_n}{(N-1)\gamma^{PI} + \gamma^{SI}} \tag{9}$$

and he solves the following maximization problem:

$$\max_{x_n \in \mathbb{R}} E\left((\widetilde{v} - \widetilde{p})\, x_n \left| \widetilde{p}, \widetilde{i}_n \right.\right) \Leftrightarrow$$

$$\max_{x_n \in \mathbb{R}} E\left(\left(\widetilde{v} - \frac{\alpha - \alpha^{PI} + \beta^{PI} \sum_{j \neq n} \widetilde{i}_j - (1 - \beta^{SI})\widetilde{S} - \overline{m} - x_n}{(N-1)\gamma^{PI} + \gamma^{SI}}\right) x_n \left| \widetilde{p}, \widetilde{i}_n \right.\right).$$

We write the first order condition for this problem and we find the optimal demand of the value-informed trader n :

$$x_n = ((N-1)\gamma^{PI} + \gamma^{SI}) \left(E\left(\widetilde{v} \middle| \widetilde{p}, \widetilde{i}_n \right) - p \right).$$

The second order sufficient condition for this maximization problem is

$$-\frac{2}{(N-1)\gamma^{PI} + \gamma^{SI}} < 0 \Leftrightarrow (N-1)\gamma^{PI} + \gamma^{SI} > 0.$$

Similarly, the supply-informed trader takes as given the strategies of the value-informed traders and in conformity with (2). The residual demand faced by him is therefore

$$p = \frac{N\alpha^{PI} + N\beta^{PI}\widetilde{v} + \beta^{PI} \sum_{n=1}^{N} \widetilde{e}_n - \overline{m} - \widetilde{S}}{N\gamma^{PI}} + \frac{y}{N\gamma^{PI}}. \tag{10}$$

The supply-informed trader solves the following maximization problem:

$$\max_{x_n \in \mathbb{R}} E\left((\widetilde{v} - \widetilde{p})\, x_n \middle| \widetilde{p}, \widetilde{i}_n \right) \Leftrightarrow$$

$$\max_{x_n \in \mathbb{R}} E\left(\left(\widetilde{v} - \frac{\alpha - \alpha^{PI} + \beta^{PI} \sum_{j \neq n} \widetilde{i}_j - (1 - \beta^{SI})\widetilde{S} - \overline{m} - x_n}{(N-1)\gamma^{PI} + \gamma^{SI}} \right) x_n \middle| \widetilde{p}, \widetilde{i}_n \right).$$

and from here we find the optimal demand of supply-informed trader

$$y = N\gamma^{PI} \left(E\left(\widetilde{v} \middle| \widetilde{p}, \widetilde{S} \right) - p \right).$$

The second order sufficient condition for this maximization problem is

$$-\frac{2}{N\gamma^{PI}} < 0 \Leftrightarrow N\gamma^{PI} > 0.$$

Since $N \geq 1$ it results $\gamma^{PI} > 0$. ■

Lemma A.3 *In a symmetric linear equilibrium for any $n = 1, ..., N$ we have*

$$\begin{aligned} E\left(\widetilde{v} \middle| \widetilde{p} = p, \widetilde{i}_n = i_n \right) &= \overline{v}\left(1 - A\left(N-1\right)\beta^{PI} - B\right) - A(\alpha - \overline{m}) \\ &\quad + (B - A\beta^{PI})\widetilde{i}_n + A\gamma\widetilde{p}. \end{aligned}$$

Proof. We can rewrite the market clearing condition (6) as

$$\widetilde{p}\gamma - \alpha + \overline{m} - \beta^{PI}\widetilde{i_n} = (N-1)\,\beta^{PI}\widetilde{v} + \beta^{PI}\sum_{j\neq n}\widetilde{e_j} - (1-\beta^{SI})\widetilde{S}. \tag{11}$$

From here it results that $\left(\widetilde{p},\widetilde{i_n}\right)$ is informationally equivalent to $\left(\widetilde{h_n},\widetilde{i_n}\right)$ where by definition $\widetilde{h_n} \equiv (N-1)\,\beta^{PI}\widetilde{v} + \beta^{PI}\sum\limits_{j\neq n}\widetilde{e_j} - (1-\beta^{SI})\widetilde{S}$. Consequently, we have $E\left(\widetilde{v}\,\middle|\,\widetilde{p}=p,\widetilde{i_n}=i_n\right) = E\left(\widetilde{v}\,\middle|\,\widetilde{h_n}=h_n,\widetilde{i_n}=i_n\right)$. Applying the projection theorem for normally distributed random variables we obtain that

$$\begin{aligned} E\left(\widetilde{v}\,\middle|\,\widetilde{h_n}=h_n,\widetilde{i_n}=i_n\right) &= \overline{v} + A\left(\widetilde{h_n} - (N-1)\,\beta^{PI}\overline{v}\right) + B\left(\widetilde{i_n}-\overline{v}\right) \\ &= \overline{v}\left(1 - A\,(N-1)\,\beta^{PI} - B\right) - A(\alpha-\overline{m}) + (B - A\beta^{PI})\widetilde{i_n} + A\gamma\widetilde{p}, \end{aligned} \tag{12}$$

where A and B are the solution of the following system of equations:

$$\begin{aligned} A &= M^{-1}(N-1)\beta^{PI}\sigma_v^2\sigma_e^2 \\ B &= M^{-1}\left[(\beta^{PI})^2\,(N-1)\,\sigma_v^2\sigma_e^2 + \left(1-\beta^{SI}\right)^2\sigma_S^2\sigma_v^2\right] \\ M &= (\beta^{PI})^2\,(N-1)\left(N\sigma_v^2+\sigma_e^2\right)\sigma_e^2 + (1-\beta^{SI})^2\sigma_S^2\left(\sigma_v^2+\sigma_e^2\right). \end{aligned} \tag{13}$$

■

Lemma A.4 *In a symmetric linear equilibrium we have*

$$E(\widetilde{v}|\widetilde{p}=p,\widetilde{S}=S) = \overline{v}\left(1-CN\beta^{PI}\right) - C(\alpha-\overline{m}) + (1-\beta^{SI})C\widetilde{S} + C\gamma\widetilde{p}.$$

Proof. We write again the market clearing condition (6) this time in order to find a pair informationally equivalent to $\left(\widetilde{p},\widetilde{S}\right)$

$$\widetilde{p}\gamma - \alpha + \overline{m} + (1-\beta^{SI})\widetilde{S} = \beta^{PI}\sum_{n=1}^{N}\widetilde{i_n}. \tag{14}$$

We define $\theta \equiv \beta^{PI}\sum\limits_{n=1}^{N}\widetilde{i_n}$. From here it results that $\left(\widetilde{\theta},\widetilde{S}\right)$ is informationally equivalent to $\left(\widetilde{p},\widetilde{S}\right)$. Consequently, $E\left(\widetilde{v}\,\middle|\,\widetilde{p}=p,\widetilde{S}=S\right) = E\left(\widetilde{v}\,\middle|\,\widetilde{\theta}=\theta,\widetilde{S}=S\right)$. Applying again the projection theorem for normally distributed random variables we obtain that

$$\begin{aligned} E\left(\widetilde{v}\,\middle|\,\widetilde{\theta}=\theta,\widetilde{S}=S\right) &= \overline{v} + C\left(\widetilde{\theta} - N\beta^{PI}\overline{v}\right) + D\widetilde{S} \\ &= \overline{v}\left(1-CN\beta^{PI}\right) - C(\alpha-\overline{m}) + (1-\beta^{SI})C\widetilde{S} + C\gamma\widetilde{p}, \end{aligned} \tag{15}$$

where

$$C = \sigma_v^2 \left(\beta^{PI} \left(N\sigma_v^2 + \sigma_e^2\right)\right)^{-1}. \tag{16}$$

■

Lemma A.5 *The coefficients $\alpha^{PI}, \beta^{PI}, \gamma^{PI}, \alpha^{SI}, \beta^{SI}, \gamma^{SI}$ are the solution of the following system of equations:*

$$\begin{cases} \alpha^{PI} = \left((N-1)\gamma^{PI} + \gamma^{SI}\right)\left(\overline{v}\left(1 - A\left(N-1\right)\beta^{PI} - B\right) - A(\alpha - \overline{m})\right) \\ \beta^{PI} = \left((N-1)\gamma^{PI} + \gamma^{SI}\right)\left(B - A\beta^{PI}\right) \\ \gamma^{PI} = \left((N-1)\gamma^{PI} + \gamma^{SI}\right)\left(1 - A\gamma\right) \\ \alpha^{SI} = N\gamma^{PI}(\overline{v}\left(1 - CN\beta^{PI}\right) - C(\alpha - \overline{m})) \\ \beta^{SI} = N\gamma^{PI}((1 - \beta^{SI})\, C \\ \gamma^{SI} = N\gamma^{PI}\left(1 - C\gamma\right) \\ M = \left(\beta^{PI}\right)^2 (N-1)\left(N\sigma_v^2 + \sigma_e^2\right)\sigma_e^2 + (1 - \beta^{SI})^2\sigma_S^2\left(\sigma_v^2 + \sigma_e^2\right) \\ A = M^{-1}(N-1)\beta^{PI}\sigma_v^2\sigma_e^2 \\ B = M^{-1}\left(\left(\beta^{PI}\right)^2 (N-1)\,\sigma_v^2\sigma_e^2 + (1 - \beta^{SI})^2\sigma_S^2\sigma_v^2\right) \\ C = \sigma_v^2\left(\beta^{PI}(N\sigma_v^2 + \sigma_e^2)\right)^{-1}. \end{cases} \tag{17}$$

Proof of Lemma A.5. In Lemma A.3 and Lemma A.4 for we have established the expressions for $E\left(\widetilde{v}\,\middle|\,\widetilde{p} = p, \widetilde{i_n} = i_n\right)$ and $E\left(\widetilde{v}\,\middle|\,\widetilde{p} = p, \widetilde{S} = S\right)$. We will use them now to find the expressions for the strategies for the value-informed agents and for the supply-informed agent.

First, since $E\left(\widetilde{v}\,\middle|\,\widetilde{p} = p, \widetilde{i_n} = i_n\right) = E\left(\widetilde{v}\,\middle|\,\widetilde{h_n} = h_n, \widetilde{i_n} = i_n\right)$ we plug (12) in (7) and we obtain that

$$\begin{aligned} x_n\left(\widetilde{p}, \widetilde{i_n}\right) &= ((N-1)\gamma^{PI} + \gamma^{SI})\left(\overline{v}\left(1 - A\left(N-1\right)\beta^{PI} - B\right) - A(\alpha - \overline{m})\right. \\ &\quad \left. + (B - A\beta^{PI})\widetilde{i_n} + (A\gamma - 1)\widetilde{p}\right). \end{aligned}$$

We identify the coefficients in the definition of the strategy of the value-informed trader n (2) and we get the following equations:

$$\begin{aligned}\alpha^{PI} &= ((N-1)\gamma^{PI}+\gamma^{SI})(\overline{v}\left(1-A\left(N-1\right)\beta^{PI}-B\right)-A(\alpha-\overline{m}))\\ \beta^{PI} &= ((N-1)\gamma^{PI}+\gamma^{SI})(B-A\beta^{PI})\\ \gamma^{PI} &= ((N-1)\gamma^{PI}+\gamma^{SI})(1-A\gamma),\end{aligned} \tag{18}$$

where A and B are given by (13).

Second, since $E\left(\widetilde{v}\,\middle|\widetilde{p}=p,\widetilde{S}=S\right)=E\left(\widetilde{v}\,\middle|\widetilde{\theta}=\theta,\widetilde{S}=S\right)$ we plug (15) in (8) and we obtain in a similar manner

$$y\left(\widetilde{p},\widetilde{S}\right)=N\gamma^{PI}\left(\overline{v}-C(\alpha-\overline{m})+(1-\beta^{SI})C\widetilde{S}+(C\gamma-1)\widetilde{p}\right).$$

We identify the coefficients in the definition of the strategy of the supply-informed trader (2) and we get the following equations:

$$\begin{aligned}\alpha^{SI} &= N\gamma^{PI}(\overline{v}\left(1-CN\beta^{PI}\right)-C(\alpha-\overline{m}))\\ \beta^{SI} &= N\gamma^{PI}(1-\beta^{SI})C\\ \gamma^{SI} &= N\gamma^{PI}(1-C\gamma),\end{aligned} \tag{19}$$

where C is given by (16).

Putting together the equations (13), (18), (16) and (19) we obtain that α^{PI},β^{PI}, $\gamma^{PI},\alpha^{SI},\beta^{SI},\gamma^{SI}$ are the solution of the above system of equations. ■

Proof of Proposition 1. The equilibrium values of the coefficients α^{PI},β^{PI}, $\gamma^{PI},\alpha^{SI},\beta^{SI},\gamma^{SI}$ are the solution of the system of equations given in the statement of Lemma A.5. After some tedious algebra and defining by

$$\delta\equiv\frac{\left(N(N-2)\sigma_v^2-\sigma_e^2\right)\left(N^2\sigma_v^2+\sigma_e^2\right)\sigma_S^2}{\left(N-1\right)\sigma_e^2},$$

we obtain the following coefficients:

$$\alpha^{PI} = \frac{\sigma_e^2\left(N\left(3N-2\right)\sigma_v^2+\left(2N-1\right)\sigma_e^2\right)\delta^{1/2}}{2N^2\sigma_v^2\left(N^2\sigma_v^2+\sigma_e^2\right)\left(N\sigma_v^2+\sigma_e^2\right)}\overline{v} + \frac{N\left(N-2\right)\sigma_v^2-\sigma_e^2}{N\left(N^2\sigma_v^2+\sigma_e^2\right)}\overline{m}$$

$$\beta^{PI} = \frac{\delta^{1/2}}{2N\left(N\sigma_v^2+\sigma_e^2\right)}$$

$$\gamma^{PI} = \frac{\left(N^2\sigma_v^2+\left(2N-1\right)\sigma_e^2\right)\delta^{1/2}}{2N^2\sigma_v^2\left(N^2\sigma_v^2+\sigma_e^2\right)}$$

$$\alpha^{SI} = \left(-\frac{\left(N-1\right)\sigma_e^2}{\left(N\sigma_v^2+\sigma_e^2\right)}\frac{N^2\sigma_v^2+\left(2N-1\right)\sigma_e^2}{2N^2\sigma_v^2\left(N^2\sigma_v^2+\sigma_e^2\right)}\delta^{1/2}\right)\overline{v} + \frac{N^2\sigma_v^2+\left(2N-1\right)\sigma_e^2}{N\left(N^2\sigma_v^2+\sigma_e^2\right)}\overline{m}$$

$$\beta^{SI} = \frac{N^2\sigma_v^2+\left(2N-1\right)\sigma_e^2}{2N\left(N\sigma_v^2+\sigma_e^2\right)}$$

$$\gamma^{SI} = -\frac{\left(N-1\right)\sigma_e^2}{\left(N\sigma_v^2+\sigma_e^2\right)}\frac{N^2\sigma_v^2+\left(2N-1\right)\sigma_e^2}{2N^2\sigma_v^2\left(N^2\sigma_v^2+\sigma_e^2\right)}\delta^{1/2}.$$

■

Proof of Corollary 2. While solving the above system we have obtained that

$$\gamma = N\gamma^{PI} + \gamma^{SI} = \frac{N^2\sigma_v^2+\left(2N-1\right)\sigma_e^2}{2N^2\sigma_v^2\left(N\sigma_v^2+\sigma_e^2\right)}\left(\frac{\left(N(N-2)\sigma_v^2-\sigma_e^2\right)\left(N^2\sigma_v^2+\sigma_e^2\right)\sigma_S^2}{\left(N-1\right)\sigma_e^2}\right)^{1/2}.$$

We study first how market depth varies when the variance of liquidity shock $\widetilde{S}$ varies. We compute the derivative $\frac{\partial\gamma}{\partial\sigma_S^2}$ and we obtain

$$\frac{\partial\gamma}{\partial\sigma_S^2} > 0.$$

Then we calculate $\frac{\partial\gamma}{\partial\sigma_e^2}$ and after somehow tedious calculations we obtain that

$$\frac{\partial\gamma}{\partial\sigma_e^2} < 0.$$

Finally, we study how the variance of liquidation value, σ_v^2 affects the market depth. We calculate the derivative $\frac{\partial\gamma}{\partial\sigma_v^2}$ and we obtain that this expression has the opposite sign to $f\left(\sigma_v^2\right)$, where

$$\begin{aligned} f\left(\sigma_v^2\right) &= N^4\left(\sigma_v^2\right)^3\left(N-1\right)\left(N^2-3N+1\right) - 3\sigma_e^2N^2\left(\sigma_v^2\right)^2\left(2N-1\right)\left(N-1\right) \\ &\quad -3\sigma_v^2\left(\sigma_e^2\right)^2N\left(2N-1\right)\left(N-1\right) - \left(\sigma_e^2\right)^3\left(2N-1\right)\left(N-1\right). \end{aligned}$$

We study this function and we obtain that the equation $f'(\sigma_v^2) = 0$,

$$f'(\sigma_v^2) = 3(N-1)N\left[\left(N^3(\sigma_v^2)^2(N^2-3N+1) - 2\sigma_e^2 N(2N-1)\sigma_v^2 - (\sigma_e^2)^2(2N-1)\right)\right],$$

has only one positive solution equal to

$$\sigma_e^2\frac{(2N-1)+(N-1)((2N-1)(N-1))^{1/2}}{N^2(N^2-3N+1)} \equiv k_l(N).$$

We obtain that $k_l(N) > \dfrac{1}{N(N-2)}$. So, it results that the function $f(\sigma_v^2)$ is decreasing for $\dfrac{\sigma_v^2}{\sigma_e^2} \in \left[\dfrac{1}{N(N-2)}, k_l(N)\right]$, and is increasing for $\dfrac{\sigma_v^2}{\sigma_e^2} > k_l(N)$. Since $f(0) = -(\sigma_e^2)^3(2N-1)(N-1)$, it results that it exists $k^*(N,\sigma_e^2)$ such that $f(k^*(N,\sigma_e^2)) = 0$. Therefore, the function $f(\sigma_v^2) < 0$ for any $\sigma_v^2 < k^*(N,\sigma_e^2)$ and is greater than 0 otherwise.

Once we have characterized the behavior of function $f(\sigma_v^2)$ we can conclude that the market depth is a increasing function of σ_v^2 if $\sigma_v^2 < k^*(N,\sigma_e^2)$ and is decreasing otherwise. ■

Proof of Corollary 3. From the market clearing condition (6) we obtain that the equilibrium price is

$$\widetilde{p} = \left(N\gamma^{PI} + \gamma^{SI}\right)^{-1}\left(\alpha + \beta^{PI}\sum_{n=1}^{N}\widetilde{i}_n - (1-\beta^{SI})\widetilde{S} - \overline{m}\right).$$

Using the formulas we have obtained for the equilibrium coefficients we can write that the equilibrium price equals to

$$\widetilde{p} = \frac{\sigma_e^2(2N-1)}{N^2\sigma_v^2 + (2N-1)\sigma_e^2}\overline{v} + \frac{N\sigma_v^2}{N^2\sigma_v^2 + (2N-1)\sigma_e^2}\sum_{n=1}^{N}\widetilde{i}_n - \frac{N\sigma_v^2(N^2\sigma_v^2+\sigma_e^2)}{(N^2\sigma_v^2 + (2N-1)\sigma_e^2)\delta^{1/2}}\widetilde{S} - \frac{2N\sigma_v^2(N\sigma_v^2+\sigma_e^2)}{(N^2\sigma_v^2 + (2N-1)\sigma_e^2)\delta^{1/2}}\overline{m}.$$

Notice that since $\widetilde{i}_n = \widetilde{v} + \widetilde{e}_n$ we can write

$$\widetilde{p} = \frac{\sigma_e^2(2N-1)}{N^2\sigma_v^2 + (2N-1)\sigma_e^2}\overline{v} + \frac{N^2\sigma_v^2}{N^2\sigma_v^2 + (2N-1)\sigma_e^2}\widetilde{v} + \frac{N\sigma_v^2}{N^2\sigma_v^2 + (2N-1)\sigma_e^2}\sum_{n=1}^{N}\widetilde{e}_n - \frac{N\sigma_v^2(N^2\sigma_v^2+\sigma_e^2)}{(N^2\sigma_v^2 + (2N-1)\sigma_e^2)\delta^{1/2}}\widetilde{S} - \frac{2N\sigma_v^2(N\sigma_v^2+\sigma_e^2)}{(N^2\sigma_v^2 + (2N-1)\sigma_e^2)\delta^{1/2}}\overline{m}.$$

Taking the expectations it results that $E(\widetilde{p}) = \overline{v} - \dfrac{2N\sigma_v^2 (N\sigma_v^2 + \sigma_e^2)}{(N^2\sigma_v^2 + (2N-1)\sigma_e^2)\,\delta^{1/2}}\overline{m}.$

■

Proof of Corollary 4. We have seen that the equilibrium price is given by (5). As a result, we can compute the variance, and after some straightforward algebra we find

$$Var(\widetilde{p}) = \frac{N^3(N-2)(\sigma_v^2)^2 + 2N^2(N-2)\sigma_v^2\sigma_e^2 - (\sigma_e^2)^2}{(N(N-2)\sigma_v^2 - \sigma_e^2)} \left(\frac{N\sigma_v^2}{N^2\sigma_v^2 + (2N-1)\sigma_e^2}\right)^2.$$

■

Proof of Corollary 5. We compute now $Var(\widetilde{v}) - Var(\widetilde{v}|\,\widetilde{p})$. Due to the normality assumptions we have that

$$Var(\widetilde{v}) - Var(\widetilde{v}|\,\widetilde{p}) = (Var(\widetilde{p}))^{-1}(Cov(\widetilde{v},\widetilde{p}))^2.$$

We calculate the covariance

$$Cov(\widetilde{v},\widetilde{p}) = \frac{(N\sigma_v^2)^2}{N^2\sigma_v^2 + (2N-1)\sigma_e^2},$$

and together with the formula for variance $Var(\widetilde{p})$ we obtained before, we plug them above to obtain

$$Var(\widetilde{v}) - Var(\widetilde{v}|\,\widetilde{p}) = \frac{N(\sigma_v^2)^2(N(N-2)\sigma_v^2 - \sigma_e^2)}{N^3(N-2)(\sigma_v^2)^2 + 2N^2(N-2)\sigma_v^2\sigma_e^2 - (\sigma_e^2)^2}.$$

■

Proof of Corollary 6. Since the demand of the value-informed agent x_n can be written as the sum of normal variables it results that x_n is also a normal variable. The mean of x_n is $\mu_n = \dfrac{(N-1)\sigma_v^2}{(N^2\sigma_v^2 + \sigma_e^2)}\overline{m}$ while the variance $\sigma_{x_n}^2$ is

$$\sigma_{x_n}^2 = Var(x_n) = \frac{(\sigma_v^2 + \sigma_e^2)\,\delta}{4N^2}\left(\frac{1}{(N\sigma_v^2 + \sigma_e^2)^2} + \frac{N}{(N^2\sigma_v^2 + \sigma_e^2)^2}\right) + \frac{\sigma_S^2}{4N^2}.$$

Then, since x_n is $\mathcal{N}(\mu_n, \sigma_{x_n}^2)$ it results that the expected volume of trade by a value-informed trader is

$$E\left(|x_n|\right) = \int_{-\infty}^{\infty} |x_n| \frac{1}{\sigma_{x_n}\sqrt{2\pi}} \exp\left(-\frac{\left(x_n-\mu_n\right)^2}{2\sigma_{x_n}^2}\right) dx_n = 2\mu_n + \left(\frac{2}{\pi}\right)^{1/2} \sigma_{x_n}^2 =$$

$$\frac{2\left(N-1\right)\sigma_v^2}{\left(N^2\sigma_v^2+\sigma_e^2\right)}\overline{m} + \left(\frac{2}{\pi}\right)^{1/2} \left(\frac{\left(\sigma_v^2+\sigma_e^2\right)\delta}{4N^2}\left(\frac{1}{\left(N\sigma_v^2+\sigma_e^2\right)^2} + \frac{N}{\left(N^2\sigma_v^2+\sigma_e^2\right)^2}\right) + \frac{1}{4N^2}\sigma_S^2\right).$$

Similarly, the quantity demanded by the supply-informed agent is a normal variable with mean $\mu_y = \frac{\left(N\sigma_v^2+\sigma_e^2\right)}{\left(N^2\sigma_v^2+\sigma_e^2\right)}\overline{m}$ and variance

$$\sigma_y^2 = Var(y) = \frac{1}{4}\sigma_S^2\left(1+\frac{\left(N-1\right)\sigma_e^2\left(N(N-2)\sigma_v^2-\sigma_e^2\right)\left(\sigma_v^2+\sigma_e^2\right)}{N\left(N^2\sigma_v^2+\sigma_e^2\right)\left(N\sigma_v^2+\sigma_e^2\right)^2}\right).$$

Then since y is $\mathcal{N}(\mu_y, \sigma_y^2)$ it results that the expected volume of trade of the supply-informed agent is

$$E\left(|y|\right) = \int_{-\infty}^{\infty} |y| \frac{1}{\sigma_y\sqrt{2\pi}} \exp\left(-\frac{\left(y-\mu_y\right)^2}{2\sigma_y^2}\right) dy = 2\mu_y + \sqrt{\frac{2}{\pi}}\sigma_y^2$$

$$= 2\frac{\left(N\sigma_v^2+\sigma_e^2\right)}{\left(N^2\sigma_v^2+\sigma_e^2\right)}\overline{m} + \left(\frac{2}{\pi}\right)^{1/2} \frac{1}{4}\sigma_S^2\left(1+\frac{\left(N-1\right)\sigma_e^2\left(N(N-2)\sigma_v^2-\sigma_e^2\right)\left(\sigma_v^2+\sigma_e^2\right)}{N\left(N^2\sigma_v^2+\sigma_e^2\right)\left(N\sigma_v^2+\sigma_e^2\right)^2}\right).$$

■

Proof of Corollary 7. Let us compute first the unconditional expected profit of the n^{th} value-informed trader.

$$\Pi_n^{PI} = E\left(\widetilde{\pi_n^{PI}}\right) = E\left(\left(\widetilde{v}-\widetilde{p}\right)\widetilde{x_n}\right).$$

Using the formulas we have obtained for $\widetilde{p}$ and $\widetilde{x_n}$ we obtain

$$\Pi_n^{PI} = \frac{\sigma_v^2\delta^{1/2}\left(N-1\right)\sigma_e^2}{2N\left(N^2\sigma_v^2+\left(2N-1\right)\sigma_e^2\right)\left(N\sigma_v^2+\sigma_e^2\right)}\left(\frac{N\left(N\sigma_v^2+\sigma_e^2\right)}{\left(N(N-2)\sigma_v^2-\sigma_e^2\right)} - \frac{\left(N-1\right)\sigma_e^2}{\left(N^2\sigma_v^2+\sigma_e^2\right)}\right)$$
$$+ \frac{\left(N-1\right)\sigma_v^2}{\left(N^2\sigma_v^2+\sigma_e^2\right)}\frac{2N\sigma_v^2\left(N\sigma_v^2+\sigma_e^2\right)}{\left(N^2\sigma_v^2+\left(2N-1\right)\sigma_e^2\right)\delta^{1/2}}\overline{m}^2.$$

Let us compute now the unconditional expected profit of the supply-informed trader

$$\Pi^{SI} = E\left(\widetilde{\pi^{SI}}\right) = E\left(\left(\widetilde{v}-\widetilde{p}\right)\widetilde{y}\right).$$

Similarly, using the formulas we have obtained for $\widetilde{p}$ and $\widetilde{y}$ we can write further

$$\begin{aligned}\Pi^{SI} &= \frac{\delta^{1/2}\left(N-1\right)\sigma_e^2\sigma_v^2}{2\left(N^2\sigma_v^2+\left(2N-1\right)\sigma_e^2\right)}\left(\frac{\left(N-1\right)\sigma_e^2}{\left(N^2\sigma_v^2+\sigma_e^2\right)\left(N\sigma_v^2+\sigma_e^2\right)}+\frac{N}{\left(N(N-2)\sigma_v^2-\sigma_e^2\right)}\right)\\ &+\frac{2N\sigma_v^2\left(N\sigma_v^2+\sigma_e^2\right)}{\left(N^2\sigma_v^2+\left(2N-1\right)\sigma_e^2\right)\delta^{1/2}}\frac{\left(N\sigma_v^2+\sigma_e^2\right)}{\left(N^2\sigma_v^2+\sigma_e^2\right)}\overline{m}^2.\end{aligned}$$

The total profits in the market are

$$\Pi = N\Pi^{PI}+\Pi^{SI} = E\left(\left(\widetilde{v}-\widetilde{p}\right)\left(\sum_{n=1}^{N}\widetilde{x_n}+\widetilde{y}\right)\right).$$

But from the market clearing condition it results that

$$\begin{aligned}\Pi &= N\Pi^{PI}+\Pi^{SI} = E\left(\left(\widetilde{v}-\widetilde{p}\right)\left(\overline{m}+\widetilde{S}\right)\right) = \\ &= \frac{N\sigma_v^2}{\left(N^2\sigma_v^2+\left(2N-1\right)\sigma_e^2\right)\delta^{1/2}}\left(\left(N^2\sigma_v^2+\sigma_e^2\right)\sigma_S^2+2\left(N\sigma_v^2+\sigma_e^2\right)\overline{m}^2\right).\end{aligned}$$

We can check and see that indeed the profits we have obtained sum up to this amount. ■

References

[1] Admati, A. and P. Pfleiderer, 1991, Sunshine Trading and Financial Market Equilibrium, Review of Financial Studies 4(3), 443-482.

[2] Back, K. and S. Baruch, 2004, Information in Securities Markets: Kyle Meets Glosten and Milgrom, Econometrica 72(2), 433-465.

[3] Bhattacharya, U. and M. Spiegel, 1991, Insiders, Outsiders, and Market Breakdowns, Review of Financial Studies 4, 255-282.

[4] Brown, D. and Z.P. Zhang, 1997, Markets Orders and Market Efficiency, Journal of Finance 52, 277-308.

[5] Caballé, J., 1992, Market Versus Limit Orders, Economic Letters 40, 339-344.

[6] Caballé, J. and M. Krishnan, 1994, Imperfect Competition in a Multi-Security Market with Risk Neutrality, Econometrica 62, 695-704.

[7] Ellis, K., Michaely, R. and M. O'Hara, 2001, The Making of a Deal Market: From Entry to Equilibrium in the Trading of Nasdaq Stocks, Working paper, Cornell University.

[8] Fishman, M. J., and F. A. Longstaff, 1992, Dual Trading in Futures Markets, Journal of Finance 47, 643-671.

[9] Foster, F.D. and S. Viswanathan, 1993, Variations in Trading Volume, Return Volatility, and Trading Costs: Evidence on Recent Price Formation Models, Journal of Finance 48, 187-211.

[10] Gennotte, G. and H. Leland, 1990, Market Liquidity, Hedging and Crashes, American Economic Review 80, 999-1021.

[11] Glosten, L. and P. Milgrom, 1985, Bid, Ask and Transaction Prices in a Specialist Market with Heterogeneously Informed Markets, Journal of Financial Economics 14, 71-100.

[12] Grossman, S. and S. Stiglitz, 1980, On the Impossibility of Informationally Efficient Markets, American Economic Review 70, 393-408.

[13] Holden, C.W. and A. Subrahmanyam, 1992, Long-Lived Private Information and Imperfect Competition, Journal of Finance 47, 247-270.

[14] Jackson, M., 1991, Equilibrium, Price Formation and the Value of Private Information, Review of Financial Studies 4, 1-16.

[15] Krishnan, M., 1992, An equivalence between the Kyle (1985) and the Glosten-Milgrom (1985) models. Economic Letters 40, 333-338.

[16] Kyle, A.S., 1985, Continuous Auctions and Insider Trading, Econometrica 53, 1315-1336.

[17] Kyle, A.S., 1989, Informed Speculators with Imperfect Competition, The Review of Economic Studies 56, 317-356.

[18] O'Hara, M., 1995, Market Microstructure Theory, Blackwell Publishers, Cambridge, Massachusetts.

[19] Madhavan, A., 1992, Trading Mechanism in Security Markets, Journal of Finance 47(2), 607-642.

[20] Palomino, F., 2001, Informational Efficiency: Ranking Markets, Economic Theory 18(3), 683-700.

[21] Rochet, J.C. and J.L. Vila, 1994, Insider Trading without Normality, The Review of Economic Studies 61, 131-152.

[22] Röell, A., 1990, Dual Capacity Trading and the Quality of the Market, Journal of Financial Intermediation 1, 105-124.

[23] Sarkar, A., 1995, Dual trading: Winners, Losers and Market Impact, Journal of Financial Intermediation 4, 77-93.

[24] Subrahmanyam, A., 1991, Risk Aversion, Market Liquidity, and Price Efficiency, Review of Financial Studies 4(3), 417-441.