Standard Error in Sampling

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The Problem: Spread in Marketing Analysis

1) CIM: reading extent of Belgian Newspapers Quarterly: 10,049 respondents: Average Spread 24%, much more for smaller newspapers (30%)-> scientific nonsense.
2) a standard error of 0.1% for the possibility that a dike is not strong enough for an eventual spring tide
3) something in between the usual standard error for marketing research is 5%.

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Example:objectives for contractors Vilnius

- 1. Cost of building management Lt/m2 *min*
- 2. Cost of common assets management Lt/m2 *min*
- 3. HVAC system maintenance cost (mean) Lt/m2 *min*
- 4. Courtyard territory cleaning (in summer) Lt/m2 *min*
- 5. Total service cost Lt/m2 *min*
- 6. Length of time in maintenance business experience in years *max*
- 7. Market share for each contractor % *max*
- 8. Number of projects per executive units/person *max*
- 9. Evaluation of management cost (Cmin / Cp) *max*

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Participants

- The largest maintenance contractors of dwellings in Vilnius were approached, of which 15 agreed
- From information of the Dwelling Owners Association, a panel of 30 owners of dwellings chosen at random agreed
- Universum around the sample

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Universum around the Sample

- Only the Vilnius population above the age of 18 has to be taken into consideration and in addition only households
- An advance payment for buying property of 15 to 30% is needed in Lithuania (Swedbank, 2012)
- In addition only 13% of the Vilnius population have a mortgage (SEB, 2013,6). From this 13% has to be excluded: existing mortgages, buying an existing property, buying a social apartment or people not interested in the location in question
- Saving rate in Lithuania was only 1.92% in 2008, which is extremely low. In 2009 there was even dissaving

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Decision Matrix: Contractors Rows Objectives Columns

	1 MIN.	2 MIN.	3 MIN.	4 MIN.	5 MIN.	6 MAX.	7 MAX.	8 MAX.	9 MAX.
a_1	0.064	0.11	0.18	0.31	0.67	12	11.75	4.6	0.83
a_2	0.06	0.14	0.37	0.12	0.5	3	0.39	0.33	0.885
a_3	0.057	0.11	0.18	0.15	0.69	12	5.25	1.47	0.935
a_4	0.06	0.12	0.10	0.15	0.57	12	7.1	2.78	0.9
a_5	0.058	0.1	0.18	0.2	0.45	12	5.56	1.39	0.9
a_6	0.071	0.3	0.18	0.26	0.82	13	26.62	5.67	0.746
a_7	0.11	0.14	0.18	0.12	0.55	5	2.82	1.2	0.483
a_8	0.058	0.18	0.37	0.19	0.61	11	9.48	3.03	0.916
a_9	0.053	0.14	0.16	0.23	0.8	11	2.23	0.8	1
a_{10}	0.07	0.26	0.29	0.2	0.7	11	13.5	9.05	0.75
a_{11}	0.12	0.2	0.09	0.2	0.81	4	4.7	1.5	0.443
a_{12}	0.071	0.28	0.18	0.28	0.73	12	2.35	0.86	0.746
a_{13}	0.078	0.2	0.18	0.3	0.76	8	5.6	3.25	0.681
a_{14}	0.056	0.14	0.18	0.12	0.5	11	2.66	1.7	0.948
a_{15}	0.12	0.14	0.09	0.21	0.56	3	0.04	0.03	0.531

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Convidence Level

- standard error se $= \sqrt{\frac{pq}{n}} = \sqrt{\frac{0.25}{30}} = 0.09$

which means 9% under or 9% above the real percentage or a Spread of 18%.

Which Method to solve the Problem?

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Which Method?

- Saw Method
- TOPSIS or VIKOR
- MULTIMOORA

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Saw Method

$$\sum_{i=1}^{i=n} w_i = 1$$

$$Max. x_j = w_1 x_{1j} + w_2 x_{2j} + ... + w_i x_{ij} + ... + w_n x_{nj}$$

- Weights → dual situation:
  1) normalization
  2) importance
- Creation of one super objective

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DECISION MATRIX

	Obj. 1	Obj. 2	……….	Obj. i	……..	Obj. n
Alternative 1	X_{11}	X_{21}	………	X_{i1}	……	X_{n1}
Alternative 2	X_{12}	X_{22}	………	X_{i2}	……	X_{n2}
…………..	…….	……..	……….	………	……	………..
Alternative j	X_{1j}	X_{2j}	………	X_{ij}	……	X_{nj}
………..	……	………	………	…….	……..	………
Alternative m	X_{1m}	X_{2m}		X_{im}		X_{nm}

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HOW TO READ THE DECISION MATRIX ?

1. HORIZONTAL WAY → weights: complicates the issue
2. VERTICAL WAY → dimensionless measures:

NOT GOOD! Each alternative per objective:

$$x_{ij}* = \frac{x_{ij}}{\sum_{j=1}^{m}}$$

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MOORA
I) Ratio System of MOORA

x_{ij} as the response of alternative j on objective I

i=1,2,…,n as the objectives
j=1,2,…,m as the alternatives.
x_{ij} = response of alternative j on objective i

$$x_{ij}* = \frac{x_{ij}}{\sqrt{\sum_{j=1}^{m} x_{ij}^2}} \quad (1)$$

$x_{ij}*$ = number without dimensions response of alternative j on objective i.

These responses belong to the interval [0; 1], but [-1; 1] remains possible.

(1) The best ratio: Brauers-Zavadskas 2006

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MOORA
I) Ratio System of MOORA

with all objectives of the same importance

$$y_j^* = \sum_{i=1}^{i=g} x_{ij}^* - \sum_{i=g+1}^{i=n} x_{ij}^* \qquad (2)$$

i = 1,2,…,g, objectives to maximized
i = g+1, g+2,…, n objectives to minimized

y_j^* **= alternative j concerning all objectives showing the final preference**

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MOORA
II) Second Part: the Method of the Reference Point

- **Which Reference Point?**
- 1) *Maximal Objective Reference Point*
- 2)*Utopian Objective Reference Point*
- 3)*Aspiration Objective Reference Point*

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Maximal Objective Reference Point

Suppose 2 points: A(100,20) and B (50,100)

Dominating coordinates →

$R_m(100;100)$

matrix: $[x_{ik}]$

Maximal Objective Reference Point

$\{r_i\} = \{r_1, r_2, \ldots, r_n\}$

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Most General Synthesis of REFERENCE POINT The MINKOWSKI MATRIX

$$Min.M_j = \left\{ \sum_{i=1}^{i=n} \left(r_i - x^*_{ij} \right)^{\alpha} \right\}^{1/\alpha}$$

<u>Rectangular Distance Metric</u> α = 1

Example: reference point (100;100)

then the points (100;0), (0;100), (50;50), (60;40), (40;60), (30;70), and (70;30) show the same rectangular distance.

A midway solution like (50;50) takes the same ranking as the extreme positions (100;0) and (0;100).

Even worse, an infinite number of points could result in the same ranking

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$$Min.M_j = \{ \sum_{i=1}^{i=n} (r_i - x_{ij}^*)^\alpha \}^{1/\alpha}$$

Euclidean Distance Metric **α = 2**

The midway solution (50;50) is ranked first with symmetry in ranking for the extreme positions:

(100;0) and (0;100); the same for (60;40) and (40;60), for (30;70) and (70;30) etc. → numerous solutions.

The same for **TOPSIS** and **VIKOR**? NO→ by making more specific by introducing significance coefficients:

$$Min.M_j = \{ \sum_{i=1}^{i=n} (s_i r_i - s_i x_{ij}^*)^2 \}^{1/2}$$

Idem for Rectangular Distance Metric

Introducing significance coefficients makes it again more complicated

The Chebicheff Min-Max function

- Using the Minkowsky metric in its most extreme ∞-norm.
- The Minkowski metric becomes a *Min-Max Metric*:The Chebicheff Min-Max function

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MOORA
II) Second Part: the Method of the Reference Point

Returning to (1) x_{ij}^*

as a function of a reference point r_i produces a matrix:

$$\left(r_i - x_{ij}^*\right)$$

r_i = Maximal Objective Reference Point (coordinates are existing)

Rectangular and Euclidean distance metrics do not satisfy consumer surplus, the Chebicheff Min-Max function does:

$$\underset{(j)}{\text{Min}}\left\{\underset{(i)}{\max}\left(\left| r_i - x_{ij}^* \right|\right)\right\} \qquad (3)$$

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However an *s* in Reference Point Theory does not change the outcome due to Chebicheff

- **Only taking *s* in the Ratio System would harm MOORA philosophy. Therefore the solution of sub-objectives**
- **<u>Example 1: direct and indirect employment</u>**
- **<u>Example 2: more importance to Pollution</u>**
- - Min. consumption of energy on basis of equivalence in kg fuel per 1000€ GNP
  - – Min. of solid emissions in kg per square km
    SO_2, NOx, dust, hydro carbon etc.
  - - Min. the others

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Superiority of MOORA

1. Composed of 2 Methods each controlling each other.
2. Stakeholders have only to choose Objectives and Alternative Solutions:

   no additional coefficients needed.

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- **CHAKRABORTY (2011)**
  Decision Making in Manufacturing
  (Springer "International Journal Advanced Manufacturing Technology)
- **Comparative Performance of some popular MODM Methods**

MODM method	Computation time	Simplicity	Mathematical calculation involved	Stability	Information Type
MOORA	Very less	Very simple	Minimum	Good	Quantitative
AHP	Very high	Very critical	Maximum	Poor	Mixed
TOPSIS	Moderate	Moderately critical	Moderate	Medium	Quantitative
VIKOR	Less	Simple	Moderate	Medium	Quantitative
ELECTRE	High	Moderately critical	Moderate	Medium	Mixed
PROMETHEE	High	Moderately critical	Moderate	Medium	Mixed

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Why not including all dimensionless measures methods? By Multiplicative form?

- Reads response matrix horizontally
- Multiplication not considering units
- Not absolute value counts but the ranking
- Ends with comparing 3 methods
- We call it then MULTIMOORA

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The Full-Multiplicative Form

$$U_j = \prod_{i=1}^{n} x_{ij}$$

***1,2,..i,..,n; n* the number of objectives; *j* = *1,2,...,m; m* the number of alternatives**

$$U'_j = \frac{A_j}{B_j}$$

g = the number of objectives to be maximized

$$A_j = \prod_{i=1}^{g} x_{ij}$$

$$B_j = \prod_{i=g+1}^{n} x_{ij}$$

n-g = the number of objectives to be minimized

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DIAGRAM OF MULTIMOORA

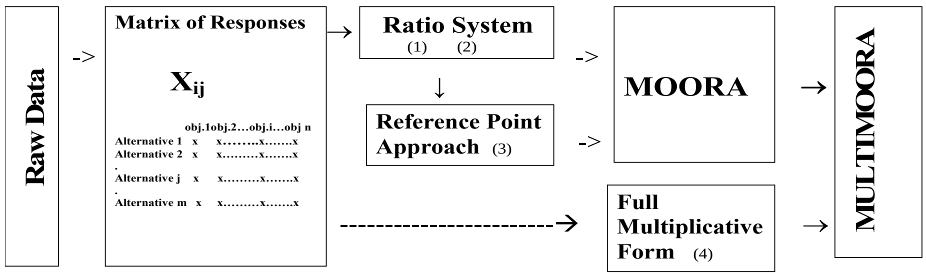

Fig. I **Diagram of MULTIMOORA**

The figures between brackets refer to the formulas on next pages.

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The Impossibility Theorem of Arrow

- 1. A deduction of an Ordinal Scale, a ranking, from cardinal data is always possible.
- 2. An Ordinal Scale can never produce a series of cardinal numbers.
- 3. An Ordinal Scale of a certain kind, a ranking, can be translated in an ordinal scale of another kind.
- In application of axiom 3 the rankings of three methods of MULTIMOORA are translated into another ordinal scale based on *Dominance*, *being Dominated, Transitivity and Equability*.
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DOMINANCE THEORY

- **Absolute Dominance:** an alternative dominates in each ranking all other alternatives, This absolute dominance shows as rankings for MULTIMOORA: (1-1-1).
- **General Dominance in two of the three methods** with a < b < c <d: is of the form: (d-a-a) is generally dominating (c-b-b)
- (a-d-a) is generally dominating (b-c-b)
- (a-a-d) is generally dominating (b-b-c) and further transitiveness plays fully.
- **Transitiveness**
- If *a* dominates *b* and *b* dominates *c* than also *a* will dominate *c*.
- **Overall Dominance of one alternative on the next one:**
  - (a-a-a) is overall dominating (b-b-b)
- **Equability**

Absolute Equability : e.g. (e-e-e) for 2 altern.

Partial Equability e. g. (5-e-7) and (6-e-3).

Circular Reasoning: Object A (11-20-14) dom. generally object B (14-16-15)

Object B. (14-16-15) dom. Object C (15-19-12) but Object C (15-19-12) dominates generally Object A (11-20-14). In such a case the same ranking is given to the three objects.

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MULTIMOORA with Spread

	RS	RP	MF	MULTIMOORA
a6	1	1	3	**1**
a4	2	5	1	**2**
a10	3	2	4	**3**
a1	4	3	2	**4**
a5	5	7	5	**5**
a3	6	8	6	**6**
a8	8	4	7	**7**
a14	7	11	8	**8**
a13	10	6	9	**9**
a9	9	13	10	**10**
a7	11	10	11	**11**
a11	14	9	12	**12**
a12	13	12	13	**13**
a2	12	14	14	**14**
a15	15	15	15	15

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Decision Matrix: Contractors Rows Objectives Columns with no Spread

Table 6 Ranking Contractors after MULTIMOORA with 9% less and 9% more for each objective

	Obj. 1		Obj.2.... Obj.3 ...Obj.4...Obj.5....Obj.6....Obj.7....Obj.8...		Obj. 9	
0.058	0.064	0.070	--	0.76	0.83	0.90
0.055	0.060	0.065	--	0.81	0.89	0.96
0.052	0.057	0.062	--	0.85	0.94	1.02
0.053	0.058	0.063	--	0.83	0.91	0.99
0.053	0.058	0.063	--	0.83	0.91	0.99
0.065	0.071	0.077	--	0.68	0.75	0.81
0.100	0.110	0.120	--	0.44	0.48	0.53
0.053	0.058	0.063	--	0.83	0.92	1.00
0.048	0.053	0.058	--	0.91	1.00	1.09
0.065	0.071	0.077	--	0.68	0.75	0.81
0.109	0.120	0.131	--	0.40	0.44	0.48
0.065	0.071	0.077	--	0.68	0.75	0.81
0.071	0.078	0.085	--	0.62	0.68	0.74
0.051	0.056	0.061	--	0.86	0.95	1.03
0.109	0.120	0.131	--	0.48	0.53	0.58

27 objectives and sub-objectives replace the 9 objectives

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Fig. 1 Original Values and Standard Errors of Objective 1

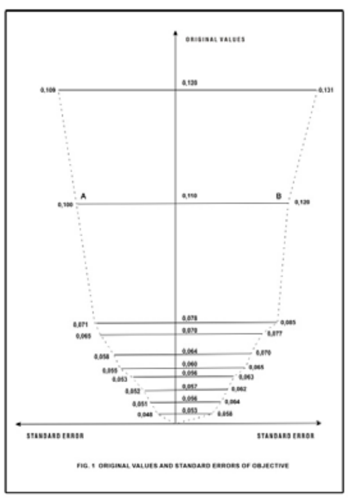

FIG. 1 ORIGINAL VALUES AND STANDARD ERRORS OF OBJECTIVE

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Fuzzy Numbers

Being a special case of the fuzzy sets, fuzzy numbers express uncertain quantities with employing them for further reasoning. Among various instances of fuzzy numbers, the triangular fuzzy numbers are often used for multi-criteria decision making. A triangular fuzzy number $\tilde{x}$ can be represented by a tripet: $\tilde{x} = (a, b, c)$, where a and c are the minimum and maximum bounds, respectively, and b is the modal value or kernel.

The following arithmetic operations are available for the fuzzy numbers:

- Addition $\oplus$: $\tilde{A} \oplus \tilde{B} = (a,b,c) \oplus (d,e,f) = (a+d, b+e, c+f)$;
- Subtraction $\ominus$: $\tilde{A} \ominus \tilde{B} = (a,b,c) \ominus (d,e,f) = (a-f, b-e, c-d)$;
- Multiplication $\otimes$: $\tilde{A} \otimes \tilde{B} = (a,b,c) \otimes (d,e,f) = (a \times d, b \times e, c \times f)$
- Division $\oslash$: $\tilde{A} \oslash \tilde{B} = (a,b,c) \oslash (d,e,f) = (a / f, b / e, c / d)$.

The vertex method can be applied to measure the distance between two fuzzy numbers. Let $\tilde{A} = (a,b,c)$ and $\tilde{B} = (d,e,f)$ be the two triangular fuzzy numbers. Then, the vertex method can be applied to measure the distance between these two fuzzy numbers:

$$d(\tilde{A}, \tilde{B}) = \sqrt{\frac{1}{3}[(a-d)^2 + (b-e)^2 + (c-f)^2]}.$$

Fuzzy MULTIMOORA

Fuzzy MULTIMOORA begins with fuzzy decision matrix $\tilde{X}$, where $\tilde{x}_{ij} = (x_{ij1}, x_{ij2}, x_{ij3})$ are aggregated responses of alternatives on objectives.

The Fuzzy Ratio System

The Ratio System defines normalization of the fuzzy numbers $\tilde{x}_{ij}$ resulting in matrix of dimensionless numbers. The normalization is performed by comparing appropriate values of fuzzy numbers:

$$\tilde{x}^*_{ij} = (x^*_{ij1}, x^*_{ij2}, x^*_{ij3}) = \begin{cases} x^*_{ij1} = x_{ij1} \Big/ \sqrt{\frac{1}{3}\sum_{i=1}^{m}\left[(x_{ij1})^2 + (x_{ij2})^2 + (x_{ij3})^2\right]} \\ x^*_{ij2} = x_{ij2} \Big/ \sqrt{\frac{1}{3}\sum_{i=1}^{m}\left[(x_{ij1})^2 + (x_{ij2})^2 + (x_{ij3})^2\right]}, \forall i, j. \\ x^*_{ij3} = x_{ij3} \Big/ \sqrt{\frac{1}{3}\sum_{i=1}^{m}\left[(x_{ij1})^2 + (x_{ij2})^2 + (x_{ij3})^2\right]} \end{cases}$$

The normalization is followed by computation of the overall utility scores, $\tilde{y}^*_i$, for each i^{th} alternative. The normalized ratios are added or subtracted with respect to the type of criteria:

$$\tilde{y}^*_i = \sum_{j=1}^{g} \tilde{x}^*_{ij} \ominus \sum_{j=g+1}^{n} \tilde{x}^*_{ij},$$

where $g = 1, 2, \ldots, n$ stands for number of criteria to be maximized. Then each ratio

$\tilde{y}^*_i = (y^*_{i1}, y^*_{i2}, y^*_{i3})$ is defuzzified: $BNP_i = \frac{y^*_{i1} + y^*_{i2} + y^*_{i3}}{3}$

BNP_i denotes the best non-fuzzy performance value of the i^{th} alternative. Consequently, the alternatives with higher BNP values are attributed with higher ranks.

Fuzzy MULTIMOORA (2)

The Fuzzy Reference Point

The fuzzy Reference Point approach is based on the fuzzy Ratio System. The Maximal Objective Reference Point (vector) $\tilde{r}$ is found according to ratios found. The j^{th} coordinate of the reference point resembles the fuzzy maximum or minimum of the j^{th} criterion $\tilde{x}_j^+$, where

$$\begin{cases} \tilde{x}_j^+ = \left(\max_i x_{ij1}^*, \max_i x_{ij2}^*, \max_i x_{ij3}^*\right), j \le g; \\ \tilde{x}_j^+ = \left(\min_i x_{ij1}^*, \min_i x_{ij2}^*, \min_i x_{ij3}^*\right), j > g. \end{cases}$$

Then the every element of normalized responses matrix is recalculated and final rank is given according to deviation from the reference point (Eq. 13) and the Min-Max Metric of Tchebycheff:

$$\min_i \left(\max_j d(\tilde{r}_j, \tilde{x}_{ij}^*) \right).$$

The Fuzzy Full Multiplicative Form

Overall utility of the i^{th} alternative can be expressed as a dimensionless number by employing:

$$\tilde{U}_i' = \tilde{A}_i \% \tilde{B}_i,$$

$\tilde{A}_i = (A_{i1}, A_{i2}, A_{i3}) = \prod_{j=1}^{g} \tilde{x}_{ij}$, $i = 1, 2, \cdots, m$ denotes the product of objectives of the i^{th} alternative to be maximized with $g = 1, \cdots, n$ being the number of criteria to be maximized.

$\tilde{B}_i = (B_{i1}, B_{i2}, B_{i3}) = \prod_{j=g+1}^{n} \tilde{x}_{ij}$ denotes the product of objectives of the i^{th} alternative to be minimized with $n - g$ as the number of criteria to be minimized. Since the overall utility $\tilde{U}_i'$ is a fuzzy number, one needs to defuzzify it to rank the alternatives (cf. Eq. 12). The higher the best non-fuzzy performance value (BNP), the higher will be the rank of a certain alternative.

Thus, the fuzzy MULTIMOORA summarizes fuzzy MOORA (i. e. fuzzy Ratio System and fuzzy Reference Point) and the fuzzy Full Multiplicative Form.

Ranking by Fuzzy MULTIMOORA after its three parts and with the application of Ordinal Dominance Theory

	Fuzzy Ratio System	**Fuzzy Reverence Point Method**	**Fuzzy Multiplicative Form**	**Fuzzy MULTIMOORA**
a6	1	1	3	1
a1	2	3	2	2
a10	3	2	4	3
a4	4	5	1	4
a5	5	7	5	5
a3	6	8	6	6
a8	8	4	7	7
a14	7	11	8	8
a13	10	6	9	9
a9	9	13	10	10
a7	12	10	11	11
a11	13	9	12	12
a12	11	12	13	13
a2	14	14	14	14
a15	15	15	15	15

Ranking Contractors after the two Possibilities

	MULTIMOORA with 18% spread	Fuzzy MULTIMOORA no spread
1	a6	a6
2	a4	a1
3	a10	a10
4	a1	a4
5	a5	a5
6	a3	a3
7	a8	a8
8	a14	a14
9	a13	a13
10	a9	a9
11	a7	a7
12	a11	a11
13	a12	a12
14	a2	a2
15	a15	a15

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RESULTS

1. The satisfaction of the eventual clients is 100% foreseen
2. Contractor a6 is preferred overall, which brings much certainty on this solution. Contrary to MULTIMOORA with 18% spread Contractor a1 is the second best
3. The winning contractor can be also a loser. *In the worst case it could be that the client asks for a 9% additional effort from the side of the contractor. Can the winning contractor not anticipate this situation? Of course he can, however with the danger that the winning contractor would become one of his colleagues.*

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